Module 10

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Module 10 Summary

I.   Capital expenditure decisions relate to the purchase of "big-ticket" long-term assets (*capital expenditures*) that the company will use through many years. The process of making capital expenditure decisions is called ***capital budgeting***.

    A.   Since capital expenditures represent major investments of the firm's funds, it is extremely important that these decisions be made carefully and properly.

    B.   Capital assets are acquired by the business to use in the production of goods and services, so capital assets should be acquired *only if they will make enough of a contribution to profits to justify the investment*. As we noted in Module 8, no company wants to invest its funds in unproductive, "wasted" assets.

II.   Capital budgeting is the term used to describe the process of identifying the need for a capital expenditure (or the availability of a profitable capital investment opportunity), determining the alternative ways that the need may be satisfied (or the investment may be undertaken), and then gathering and analyzing the information needed to choose among the alternatives.

    A.   The information needed to evaluate alternatives must be supplied by financial, marketing, and production specialists in the organization. Therefore, capital budgeting is a "business-wide" process that requires input from all facets of the organization.

        1.   The acquisition of capital usually affects both short-term and the long-term budgets and plans.

        2.   The *capital expenditure budget* is a component of the master budget that lists the capital assets to be purchased in the budget period. Long-range budgets may call for the acquisition of capital assets in the future, but the actual analysis and decision will not occur until it is time to purchase the asset. Therefore, it is only at that time that the estimated effects of the asset upon revenues and costs can be incorporated into the more detailed, shorter-term budgets

    B.   The role of the management accountant in the capital budgeting process is related most directly to the evaluation of proposals (gathering data and applying one of the evaluation methods discussed below), reporting results, and to conducting *post-audits* (follow-up studies to see how valid the estimates used to make the decision were, so that errors can be avoided in the future).

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Note: Compound interest calculations are fundamental to any discussion of capital budgeting (the subject of this chapter). Therefore, before we look into capital budgeting topics we must build a strong foundation regarding the use of these compound interest tools. The following section begins with an introduction to compound interest and leads on into its application in capital budgeting. In today's world, financial analysts use either *Microsoft Excel* or *financial calculators* when working with compound interest calculations, and for this reason many of the examples in this module illustrate the use of a *Texas Instruments Business Analyst II Plus* financial calculator.

While it is not required, you are encouraged to purchase this calculator and learn to use it as you complete this module. It makes calculations easier, your answers will not have rounding errors, and you will be learning a skill that will serve you well in the workplace. However, *in order to arrive at the answer that Cengage will mark as correct when you work the assigned homework problems, you will need to use the compound interest tables that are given to you when you open the assigned problem in Cengage*.
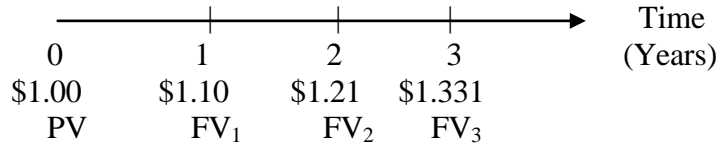
Refer to the supplement at the end of this module for comprehensive instructions regarding the setup and use of the *Business Analyst* calculator. If you would like to look at a couple of video tutorials, the links below will take you to some good user-submitted lessons on YouTube. Several other calculator tutorials may be also be played on YouTube, and if you click either of the links below you will see them listed on the right side of the YouTube screen.

http://www.youtube.com/watch?v=OxJ9fZt5Y5Y
http://www.youtube.com/watch?v=nScQsMmohZ0

III.   **Compound interest** computations are applied in making many business (and personal) decisions. Compound interest differs from **simple interest** in that *interest is earned on accumulated interest*. This does not happen with simple interest.

   A.   For example, a $1 deposit at a 10% annual interest rate earns interest of $.10 through the first year. This is true whether the 10% is a simple rate or a compounded rate. If interest compounds, though, $.11 is earned in the second year ($1.10 x 10%) while only $.10 would be earned at a 10% simple interest rate. This occurs because an additional penny of interest would be earned on the accumulated first year's interest if interest is compounded, while interest is earned only on the original deposit under the simple interest method.

   B.   In the example above, $1 grew to $1.10 ($1 + $1x10%) after one year at 10% compounded annually. The $1.10 grew to $1.21 ($1.10 + $1.10x10%) after one additional year. The $1.21 will grow to $1.331 ($1.21 + $1.21x10%) through the following year. Therefore, the $1 *principal* amount becomes worth $1.10 after one

year, $1.21 after two years, and $1.331 after three years. On a "time line" these amounts appear as follows:

```
                  +         +         +           Time
   0        1         2         3         (Years)
 $1.00     $1.10     $1.21    $1.331
  PV       FV₁       FV₂      FV₃
```

Since the $1 principal amount is the value of the deposit at the present time, it is referred to as the **present value**, or **PV**. The $1.10 is the value after one year in the future, and it is referred to as the **future value** after one year, or FV_1. Likewise, the *future value* after two years is $1.21, and the *future value* after three years is $1.331.

C.   The types of business applications in which future values must be calculated for a present value often involve planning for funds flows and financing requirements. A few examples will clarify these kinds of applications.

1.   LMN Company's materials supplier has been raising prices by 10% per year, on average, over the past few years. The present materials cost to LMN is $1 per pound. LMN's management has decided that LMN will have to raise the price of its product if the materials price exceeds $1.20 per pound. Will this happen in the next three years?

> *Answer: At 10% per year, the cost will become $1.10 after one year, $1.21 after two, and $1.33 after three years. Therefore, yes, the cost will exceed $1.20 in two years, and LMN should plan on raising its product price at that time.*

2.   ABC Company is considering the expansion of its building to handle increasing sales volume. Sales amounted to $100,000 last year, and it is thought that, given the present size of the building, a sales level of $120,000 is the maximum attainable. Sales have been growing at 10% per year over the past few years. At this rate of growth, what will the level of sales be in three years?

> *Answer: At 10% per year, sales will amount to $133,100 in three years. In fact, in two years sales will be $121,000. Therefore, the company should expand the building this year to support increasing sales levels in the following years.*

3.   XYZ Company has identified a financing requirement that will occur in 3 years: the purchase for $120,000 of a new plant asset to replace an older one which is becoming obsolete. At present, the company has $100,000 invested in marketable securities which provide a return of 10% per year. Will the company have enough cash to purchase the asset in three years?

> *Answer: At 10% per year, the investment will grow to $110,000 after one year, $121,000 after two years, and $133,100 after three. Therefore, ample funds should be available to purchase the asset.*
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IV.   Calculating Future Values from a Given Present Value.

     A.   Calculating the future value of a present amount is not complicated, but the process can become very burdensome if several years are involved. For example, suppose we are interested in calculating the future value of our original $1 present value after 100 years at 10% per year. We could continue the process used above, finding the first year's future value and then using it to determine the second year's, which could be used to find the third and so on, but this would require 100 computations. Fortunately, a much easier method exists. The procedure outlined above may be reduced to a simple formula that can be used with any number of years and any interest rate. Future values may be obtained by multiplying the present amount by a "factor" obtained by adding one to the decimal interest rate ("i"), and raising the sum to a power equal to the number of years involved ("n"):

$$FV_n = PV\,(1+i)^n$$

Instructor's Lecture Notes:

You are not responsible for memorizing this or any other compound interest formula, and it is not necessary that you be able to use algebra to solve them. This course requires only that you be able to use the compound interest functions that are built into the *Texas Instruments BA II Plus* financial calculator to solve compound interest problems. However, to be thorough in our coverage of this topic, the section below details the calculator keystrokes required to apply the formula above in order to solve for a future value.

This is not a requirement of the course and you will not be tested on this, but if your calculator has a key marked $\boxed{y^x}$ you can use the above formula to calculate future values by using the following procedure. To solve for FV_1 in the earlier example, enter the number "1" in the display; press + ; enter the decimal interest rate, ".10"; press = ; press the y^x key; enter the number of periods, "1"; and then press = again. The answer displayed on the screen should be "1.10." This is the value of $(1+i)^n$ so all that is left to do is to multiply this by the present value amount ($1) to calculate the value of FV_1, $1.10. Applying the formula to the other examples, where $1 was left for two and three years at 10% per year, will produce the following FV amounts:

$$FV_2 = P(1+i)^2 = \$1(1+.10)^2 = \$1(1.10)^2 = \$1 \times 1.21 = \$1.21$$

$$FV_3 = P(1+i)^3 = \$1(1+.10)^3 = \$1(1.10)^3 = \$1 \times 1.331 = \$1.331$$

And, to illustrate the power of compounding interest over time:

$$FV100 = P(1+i)^{100} = \$1(1+.10)^{100} = \$13,780.61$$

B.   ***Compound Interest Tables***. Before calculators and computers became commonplace, *compound interest tables* were used to look up future value "factors." These tables simply list the values of $(1+i)^n$ for various interest rates and various numbers of periods. To illustrate the common method of presentation, an abbreviated future value table is presented below.

Table 1 -- Future Value Table							
Interest Rate (i)							
Pd. (n)	1%	2%	4%	6%	8%	10%	14%
1	1.0100	1.0200	1.0400	1.0600	1.0800	1.1000	1.1400
2	1.0201	1.0404	1.0816	1.1236	1.1664	1.2100	1.2996
3	1.0303	1.0612	1.1249	1.1910	1.2597	1.3310	1.4815
4	1.0406	1.0824	1.1699	1.2625	1.3605	1.4641	1.6890
5	1.0510	1.1041	1.2167	1.3382	1.4693	1.6105	1.9254

Instructor's Lecture Notes:
Note that the values in the table above are rounded to 4 decimal places. When working problems with large amounts (hundreds of thousands of dollars, or millions), it is likely that the answer derived from the table values will be different from the answer obtained from Excel or a financial calculator. This will be caused by the rounding error that occurs when table values are used. The calculator and the spreadsheet do not round anything, and so are not subject to rounding errors.

Here's an example: To use the table to find the future value of a $100 present value after 3 years at 10% compounded annually, it is only necessary to look down the "10%" column until the "year 3" row is reached. The factor value is 1.331, which is the value of $(1+.10)^3$. Therefore, the future value at the end of year 3 is $133.10 ($100 x 1.331). You may wish to practice use of the table by working the following problems, comparing your answers to the check figures given. The actual, un-rounded future values are shown in blue, following the table-calculated amounts.

What is the future value of $10 after 3 years at 10%? (*$13.31 - $13.31*)

What is the future value of $1,000 after 2 years at 8%? (*$1,166.40 - $1,166.40*)

What is the future value of $10,000 after 5 years at 14%? (*$19,254 - $19,254.15*)

What is the future value of $1,000,000 after 5 years at 14%? (*$1,925,400 - $1,925,414.58*)

C.   ***Using the Texas Instruments BA II Plus to Solve for Future Values***. Formulas are difficult to solve accurately and quickly, and two problems arise when using a compound interest factor table. Not all the rates are given in the tables, and factor values are rounded (which can produce rounding errors in the calculations). With the advent of financial calculators and personal computers, the use of compound interest tables and formulas has become a rarity in day-to-day business operations. For these reasons, we will illustrate the use of the *Texas Instruments BA II Plus* financial calculator in the example presented in the module and we encourage you to invest in one and learn to use it.

1.   Financial calculators have rows of special keys that allow for easy entry of data into the calculator's memory and for speedy, accurate calculation of future values (as well as other application results). On the BAII Plus, the keys we'll be using most are:

     $\boxed{N}$ $\boxed{I/Y}$ $\boxed{PV}$ $\boxed{PMT}$ $\boxed{FV}$

2.   The boxed section below illustrates the use of the *BA II Plus* calculator in solving for future values.

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Note to the student:
You are not responsible for being able to use your BA II Plus calculator to solve compound interest problems, and the assignments will require you to use the table values that are given in Cengage. The exam questions can be worked either with the table values or directly on your calculator.

Refer to the supplement at the end of this module for comprehensive instructions regarding the setup and use of the Business Analyst calculator. The section below details the way that the BA II Plus calculator is used to solve for future values.

Using the Business Analyst Calculator to Solve for Future Values
Before the calculator can be used to solve compound interest problems, all the calculator's memory functions should be cleared so that you won't have data stored during previous operations interfering with the present application. Do this by pressing $\boxed{2nd}$ $\boxed{CLR\ TVM}$. This clears all the memory registers, and you should get into the habit of doing this every time you begin work on a new problem.

Entering Data:
Financial calculators are easily distinguished from other types by a row (or rows) of "financial function" keys. On the *BA II Plus*, these keys appear as follows:

$\boxed{N}$ $\boxed{I/Y}$ $\boxed{PV}$ $\boxed{PMT}$ $\boxed{FV}$

To solve for FV_1 in the earlier example, enter the number of periods, "1," in the display and press $\boxed{N}$; enter the interest rate as a whole number, "10," and press $\boxed{I/Y}$; then enter the present value amount, "1," and press $\boxed{PV}$. There is nothing to enter for $\boxed{PMT}$. We now need to tell the calculator to use the data entered to solve for the future value. Do this by pressing the $\boxed{CPT}$ key

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and then $\boxed{FV}$. The answer displayed on the screen should be "1.10." This is the future value of $1 left for 1 year at 10%. You should practice your calculator skills by entering the information from the other examples, where $1 was left for 2, 3, and 100 years at 10% per year, into the calculator. You should obtain the same results that we did before when we used the formula:

- $FV_2 = PV(1+i)^2 = \$1(1+.10)^2 = \$1(1.10)^2 = \$1 \times 1.21 = \underline{\$1.21}$
- $FV_3 = PV(1+i)^3 = \$1(1+.10)^3 = \$1(1.10)^3 = \$1 \times 1.331 = \underline{\$1.331}$
- $FV_{100} = PV(1+I)^{100} = \$1(1+.10)^{100} = \underline{\$13,780.61}$

IV.   Calculating a Present Value from a Given Future Amount.

The business applications for computation of a future value are straightforward and easily understood. Situations do arise, however, where a future value is known and a present value needs to be determined. In fact, the capital budgeting techniques presented later in this module are all based on present value computations.

A.   The terminology changes when present values are calculated. Interest is ***compounded*** when a future value is calculated from a present value, but interest is ***discounted*** from a future value to determine a present value.

B.   ***Present Value Formula and Tables***. You are not responsible for memorizing this formula nor for solving it algebraically, but it is presented here in order to be as complete as possible in our coverage. Returning to the formula for calculating FV_n above, if "n," "i," and "FV_n" are known, we may easily manipulate our future value formula to solve for "PV:"

If FV_n $=$ $PV\,(1+i)^n$

Then PV_n $= \dfrac{FV_n}{(1+i)^n} = FV_n \times \dfrac{1}{(1+i)^n}$

1.   *Remember, you are not required to work with the formulas in this course.* This formula may be solved on the calculator, and the value of $1/(1+i)^n$ can then be multiplied by the FV amount to calculate the present value.
2.   Another special table may be used to look up the value of $1/(1+i)^n$. A ***present value table*** similar to the one for future values presented above (but constructed for present value instead of future value factors) is provided in the e-book and in your Cengage assignments. You must use the table values to arrive at an answer that Cengage will mark as correct.
3.   A present value table is used in the same way as the future value table. Given an "n", "i", and "FV", one merely reads down the appropriate interest column until the correct "n" is reached. The factor listed in the table is multiplied by the "FV" to obtain the present value.

Table 2 -- Present Value Table							
Interest Rate (i)							
Pd. (n)	1%	2%	4%	6%	8%	10%	14%
1	0.9901	0.9804	0.9615	0.9434	0.9259	0.9091	0.8772
2	0.9803	0.9612	0.9246	0.8900	0.8573	0.8264	0.7695
3	0.9706	0.9423	0.889	0.8396	0.7938	0.7513	0.6750
4	0.9610	0.9238	0.8548	0.7921	0.7350	0.6830	0.5921
5	0.9515	0.9057	0.8219	0.7473	0.6806	0.6209	0.5194

Instructor's Lecture Notes:

Note that the values in the table above are rounded to 4 decimal places. When working problems with large amounts (hundreds of thousands of dollars, or millions), it is likely that the answer derived from the table values will be different from the answer obtained from Excel or a financial calculator. This will be caused by the rounding error that occurs when table values are used. The calculator and the spreadsheet do not round anything, and so are not subject to rounding errors.

Here's an example: To use the table to find the present value of $100 three years in the future at 10% compounded annually, it is only necessary to look down the "10%" column until the "year 3" row is reached. The factor value is .7513, which is the value of $1/(1+.10)^3$. Therefore, the present value of $100 three years in the future is $75.13 ($100 x .7513). You may wish to practice by working the following problems, comparing your answers to the check figures given. The actual, un-rounded present values are shown in blue, following the table-calculated amounts.

What is the present value of $10 in 3 years at 10%? (*$7.51 - $7.51*)

What is the present value of $1,000 in 2 years at 8%? (*$857.30 - $857.34*)

What is the present value of $10,000 in 5 years at 14%? (*$5,194 - $5,193.69*)

What is the present value of $1,000,000 in 5 years at 14%? (*$5,194,000 - $5,193,686.64*)

C.   ***Using the Texas Instruments Business Analyst II Plus Calculator to Solve for Present Values***. The boxed section below illustrates the use of the BA35 calculator in solving for present values.

Note to the student: The section below details the keystrokes that are used to solve for present values for single future value amounts.

Using the BA II Plus to Solve for Present Values

Clear the calculator's memory by pressing 2nd CLR TVM .

To solve for PV instead of FV, you need only enter the number of periods in the display and press N ; enter the interest rate as a whole number and press I/Y ; then enter the future value amount and press FV . There is still nothing to enter as a PMT. We now need to tell the calculator to use the data entered to solve for the present value. Do this by pressing the CPT key and then PV . The answer displayed on the screen is the present value. You should practice your calculator skills by entering the information from the examples below and calculating the present value.

- What is the present value of $133.10 in 3 years at 10%? ($100)
- What is the present value of $1,166.40 in 2 years at 8%? ($1,000)
- What is the present value of $19,254 in 5 years at 14%? ($10,000)

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D. **Present Value Applications**. The business situations in which present values need to be calculated are not intuitively obvious ones. The following example illustrates one such application.

1. ABC Company has identified a financing requirement, the purchase in three years of equipment costing $133,100. The company expects that it can earn an interest rate of 10% per year on its investment in marketable securities. How much money must the firm invest at present in marketable securities in order to accumulate the needed $133,100?

*Answer: It is the **present value** of $133,100 that will grow to $133,100 over three years at 10%. Entering the information into the calculator and solving for the present value produces an answer of $100,000. Therefore, the firm must invest $100,000 now in order to accumulate the needed $133,100. This is so because $100,000 is the present value of $133,100 discounted three years at 10% per year.*

2. As shown above, one type of present value application pertains to the determination of the amount that must be deposited at the present time so that a given amount becomes available at a particular time in the future. However, the present value application that we are most concerned with in this module involves the evaluation of investment opportunities using the **net present value** method.

V. **Calculating Net Present Value (NPV**). Calculating the net present value of an investment opportunity is a relatively straightforward process. We are interested in calculating the amount that *should* be paid for the asset we are evaluating in order to earn some minimum rate of return from the investment. Once this *theoretical value* is determined it can be compared to the actual cost of the asset to decide whether the investment should be made.

A.   ***NPV and Stock Investments***. We normally think of stocks and bonds as investments, and companies do often invest in these securities. How do investors choose which securities to buy and which to avoid?

Example:

Suppose ABC Company wants to earn at least 10% on any investment it makes. This 10% rate is called the company's ***cost of capital*** or ***hurdle rate***. The company is considering a short-term investment in XYZ Corporation's stock. The stock is currently selling for $50 per share. If ABC buys the stock, it will be held for exactly one year and then sold. A dividend of $1 per share is expected to be received at the end of the year, and the stock will then be sold for a predicted price of $59 per share. Should the investment be made?

> *Answer: If the stock is purchased, $60 per share is expected to be received in one year ($59 from the sale and $1 from the dividend). The company wants to earn 10% on the investment. Therefore, the amount that should be paid is equal to the present value of the $60 that will be received in one year, discounted at 10% per year. We may either enter these numbers (FV = $60, N = 1, I = 10) into the calculator, or use the factors from Table 2, to solve for the present value. This results in a value of $54.55, the amount that, if paid for the stock today, will result in a rate of return of exactly 10% for the one-year holding period.*

This theoretical value for the investment compares very favorably with the actual cost. In fact, the stock can be purchased for $4.55 below the price that provides a 10% rate of return (i.e., $54.55). Since the company will earn more than 10% if it buys the stock for only $50, the investment should certainly be made. The $4.55 favorable difference is the stock investment's ***net present value***, or ***NPV***:

$$\text{Theoretical Value} = \$54.55$$
$$\text{Actual Cost} = \underline{(50.00)}$$
$$\text{Net Present Value} = \$\,\underline{4.55}$$

This illustration introduces a major concept regarding NPV: ***If an investment's NPV is positive*** (the theoretical value is greater than the actual cost), ***then the rate of return expected to be earned from the investment is greater than the hurdle rate and the investment should be undertaken***.

Video lecture. Click the links below to play two video lectures that discuss present value and decision-making. Why do we need to calculate present values when we make investment decisions? What does "Net Present Value" represent, and how is the "hurdle rate" determined?

[Present Values and Decision Making](#)

[The Meaning of NPV and Determination of the Hurdle Rate](#)

B.   ***NPV and Capital Asset Investment***. Capital assets are physical assets that are used to produce goods and services. Companies must also invest funds in these assets in order to carry on operations and produce profits. How does a company choose between capital assets to determine which ones it should buy?

Here's an Example. ABC company can buy a machine for $5,000. The asset's useful life is 5 years, and a $10,000 cash profit is expected to be received at the end of the fifth year of its use. No other returns are expected. Should the firm purchase the asset?

Returning to the discussion in the previous example, we know that the firm will make the investment if a rate of return of at least 10% per year can be earned. The future return of $10,000 in five years is known, and the required rate of return of 10% is also known. We next ask the question, "what amount should the firm invest so that 10% is earned on the investment?" This is simply the present value of the $10,000 future amount discounted at 10%, or $6,209. This $6,209 is the "fair value" (or theoretical value) of the asset.

In this case, however, the firm needs to invest only $5,000 and it will still have $10,000 in accumulated value in five years. Obviously, then, the true rate of return provided by this asset is greater than the 10% required. The $5,000 actual cost of the asset is another "bargain" price, since the company could afford to pay up to $6,209 and still earn the required 10% rate of return. The amount of the bargain is equal to the asset's positive NPV of $1,209 ($6,209 – 5,000):

      Theoretical Value = $6,209
      Actual Cost =   (5,000)
      Net Present Value = $1,209

Note: Had the asset's price been, say, $7,000 instead of $5,000, the NPV calculated above would have been negative:

      Theoretical Value = $6,209
      Actual Cost =   (7,000)
      Net Present Value = $ (791)

In this case, since the company should pay $6,209 in order to earn 10%, payment of $7,000 for the same asset results in a rate of return below 10%. The negative NPV indicates that the asset should not be purchased.

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C.   ***NPV Decision Rules***. The interpretation of the NPV calculated for an investment can be summarized as follows:

1. ***Any time an investment's NPV is positive, the investment should be made***. A positive NPV means the company will earn a rate of return that is *greater* than what it requires from the investment.
2. ***If the NPV is negative***, the company will earn *less* than the rate of return required and ***the investment should be rejected***.
3. ***If the NPV is zero***, the company will earn *exactly* the rate of return it requires, so ***the investment should be made***.

VI.   NPV and Multiple Returns (Ordinary Annuities).

All the calculations to date have dealt with single sums. However, most plant assets promise to provide a ***series of returns*** rather than a single one. The net present value method can be applied in these cases as well. It is only necessary to add up all the present values of each of the future cash returns and then compare the total present value to the actual cost.

A.   Consider Asset A, a capital investment that costs $2,450, and promises to provide returns of $1,000 per year for the next three years. The required rate of return is again 10%. To apply the net present value method we need to first determine the present value of all the future returns, discounting at 10%. This can be done by simply adding the present values of each of the future returns as shown below:

Period	Future Return FV_n	Present Value (at 10%)
1	1,000	909.10
2	1,000	826.40
3	1,000	751.30
Total		2,486.80

Therefore, the total present value of the asset's returns is $2,486.80, and the net present value is a positive $36.80 ($2,486.80 - $2,450). The asset should be purchased.

B.   ***Using Formulas and Tables to Deal With Multiple Cash Flows***. A formula exists for the calculation of present values for a series of future returns (called an ***annuity of payments***). A table of compound interest factors for annuities is presented below. ***Caution:*** This table can only be used when the amounts of the payment in the series are all equal. If the amounts vary, you must calculate each payment's present value individually, and then add them all up. Also, remember that you must use the table values in your calculations when you do the assigned problems in Cengage.
   1.   The annuity present value table is used in the same way as the present value table described earlier. Given an "n", "i", and payment amount ("Pmt"), one merely reads down the appropriate interest column until the correct "n" is

reached. The factor listed in the table is multiplied by the "Pmt" to obtain the present value of the whole series of payments.

Table 3 -- Present Value Table for Annuities of Payments							
Interest Rate (i)							
Pd. (n)	1%	2%	4%	6%	8%	10%	14%
1	0.9901	0.9804	0.9615	0.9434	0.9259	0.9091	0.8772
2	1.9704	1.9416	1.8861	1.8334	1.7833	1.7355	1.6467
3	2.9410	2.8839	2.7751	2.673	2.5771	2.4869	2.3216
4	3.9020	3.8077	3.6299	3.4651	3.3121	3.1699	2.9137
5	4.8534	4.7135	4.4518	4.2124	3.9927	3.7908	3.4331

Here's an example: To use the table to find the present value of three future payments of $100 each at 10% compounded annually, it is only necessary to look down the "10%" column until the "year 3" row is reached. The factor value is 2.4869, which is the value of the annuity compound interest formula. Therefore, the present value of three annual $100 payments is $248.69 ($100 x 2.4869). You may wish to practice by working the following problems, comparing your answers to the check figures given. The actual, un-rounded present values are shown in blue, following the table-calculated amounts.

What is the present value of 3 future annual payments of $10 each at 10%? (*$24.87 - $24.87*)

What is the present value of 2 future annual payments of $1,000 each at 8%? (*$1,783.30 - $1,783.30*)

What is the present value of 5 future annual payments of $10,000 each at 14%? (*$34,331 - $34,330.81*)

C. ***Using the Business Analyst II Plus Calculator to Deal With Multiple Cash Flows***. Let's first see how multiple cash flows can be handled on the BA II Plus calculator. Remember, though, that you must use the table values in your calculations when you do the assigned problems in Cengage.

Note to the Student:

You are not responsible for being able to use your *BA II Plus* calculator to solve compound interest problems. Refer to the supplement at the end of this module for comprehensive instructions regarding the setup and use of the Business Analyst calculator. The section below details the way that the calculator is used with a ***series of cash flows*** (called an ***annuity*** of payments).

Using the Business Analyst Calculator to Solve for the NPV with Multiple Cash Flows:

Clear the calculator's memory functions by pressing **2nd** **CLR TVM** .

To solve the problem above, where $1,000 was going to be received each year for 3 years, we must enter again enter "3," the number of periods, and press ☐ **N** ; enter "10" as the interest rate and press **I/Y** ; and then, instead of entering a future value amount, enter the amount of the *payment* ($1,000) and press **PMT**. Since we have only payments to deal with, there is nothing to enter as a ☐ **FV** . As we did before, solve for the present value by pressing the **CPT** key and then **PV** . The answer displayed on the screen is $2,486.80, the same present value calculated earlier. The NPV is:

Theoretical Value = $2,486.80
Actual Cost =   (2,450.00)
Net Present Value = $ 36.80

You should practice your calculator skills by entering the information from the examples below and calculating the present value.

- What is the present value of a series of 10 payments of $100 each, discounted at 15%? ($501.88)
- What is the present value of a series of 4 payments of $5,000 each, discounted at 5%? ($17,729.75)
- What is the present value of a series of 50 payments of $50 each, discounted at 14%? ($356.63)

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C.   ***Unequal Multiple Returns***. If the future returns provided by the asset are not all equal, then an ordinary annuity does not exist and the **PMT** cannot be used to calculate the total present value. In these cases, each future return's present value must be calculated individually and then summed to determine the overall present value. This is precisely what we did with Asset A earlier when we used the calculator and its **PMT** key.

1.   For example, assume that Asset B costs $5,000 and promises to produce returns of $1,000, $2,000, and $3,000 over the next three years. Since an ordinary annuity of payments does not exist, the present value of Asset B's future cash returns will be have to be calculated as follows:

Period "n"	Return FV_n	Present Value
1	1,000	909.10
2	2,000	1,652.80
3	3,000	2,253.90
Total		4,815.80

2.   Asset B's NPV is a negative $184.20 ($4,815.80 total present value - $5,000 cost), so the asset, in this case, should be rejected.

VII. **Internal Rate of Return (IRR).** The net present value method is a theoretically valid method to apply in making capital investment decisions. Another equally valid method, the ***internal rate of return***, or ***IRR***, method, is also based upon compound interest calculations. It will always produce the same decision result as NPV (i.e., either to accept or to reject the investment), but there is one major difference. While a positive NPV indicates that a rate of return greater than the rate required will be earned, it does not tell the user what that rate of return is. *The IRR method calculates the actual rate of return provided by the investment (the internal rate of return), and this rate is then compared to the required rate (the hurdle rate) in order to decide whether the investment should be made.*

Example: NPV and IRR.

Assume a company has a hurdle rate of 10%. An opportunity comes along that promises to provide one cash return of $1,200 in one year. The cost is $1,000.

NPV. To apply NPV, we would determine the present value of the $1,200 future amount, discounting at 10%, and then subtract the $1,000 price:

> Theoretical Value = $1,090.91
> Actual Cost =   (1,000.00)
> Net Present Value = $ 90.91

The positive NPV indicates that the company should accept the investment because a rate of return greater than 10% will be earned from it. However, there is no indication of what the rate of return earned might be.

IRR. If the IRR method is utilized, we would begin by calculating this rate of return (the investment's internal rate of return). Since the $1,000 present value grows to a future value of $1,200 in one year, it is obvious that a rate of return of 20% is earned through the year. This 20% IRR can then be compared to the 10% required rate of return in order to determine whether the investment should be accepted. Since it is greater than the 10% required, the investment should be accepted. If the IRR were below the required rate, the investment would be rejected.

    A.   ***Using tables to calculate the IRR***. In the example above, the situation was simple enough that the IRR could be determined without help from the calculator. If more than one year or if multiple cash flows are involved, the process is much more difficult. In fact, before the advent of calculators and computers it was often impractical to even attempt a solution. Financial calculators and personal computers make the IRR method as simple to apply as NPV, but you will need to use the table values to calculate the IRR in Cengage.

1. The general approach that is used when tables are involved is to (1) use the given information to determine what the table value must be, and then (2) refer to the table to find the interest rate that corresponds to that value.

2. For example, in the illustration above we know that if the asset is purchased for the asking price, the actual present value amount is $1,000 and the future payment amount (with "n" = 1) is $1,200. Since the present value is equal to the payment times the table value, we can solve for the table value as follows:

If $PV_{annuity}$ =   ***Pmt x (Table Value)***

Then $\dfrac{\textbf{\textit{Table Value}}}{} = \dfrac{\textbf{\textit{PV}}_{annuity}}{\textbf{\textit{Pmt}}}$ =   ***$1,000 / $1,200 = .8333***

We now refer to Table 3 (see below), and we see that the value .8333 lies about half way between the values that correspond to interest rates of 10% and 14%.

Table 3 -- Present Value Table for Annuities of Payments							
Pd. (n)	Interest Rate (i)					*The value .8333 lies about half way between these two values.*	
	1%	2%	4%	6%	8%	10%	14%
1	0.9901	0.9804	0.9615	0.9434	0.9259	0.9091	0.8772

Therefore, we estimate that the interest rate that corresponds to the value .8333 must lie about half way between 10% and 14%. The internal rate of return is about 12%.

B.   ***Using the Business Analyst Calculator to Solve for the IRR***. In the example above, the situation was simple enough that the IRR could be determined without help from the calculator. If more than one year or if multiple cash flows are involved, the process is much more difficult. In fact, before the advent of calculators and computers it was often impractical to even attempt a solution. Now, financial calculators and personal computers make the IRR method as simple to apply as NPV.

Note. Remember that you are not responsible for being able to use your BA II Plus calculator to solve compound interest problems. The section below details the way that the Business Analyst calculator is used to solve for interest rates (the IRR for an investment). See the supplement at the end of the module for additional details.

<u>*Using the Business Analyst Calculator to Solve for the IRR*</u>:

When NPV was introduced in the section above, an example was used in which an asset with a cost of $2,450 was going to provide annual returns of $1,000 per year for 3 years. The hurdle rate was 10%, and a positive NPV of $36.80 was calculated for the asset. This told us that a rate of return greater than 10% would be earned from the investment, but it did not tell us what that rate was.

To calculate the internal rate of return that will be earned from the investment first clear the calculator's memory by pressing **2nd** **CLR TVM** . Enter the number of periods (3) and press **N** ; enter the actual cost ($2,400) as the present value **PV** ; then enter the amount of the payment ($1,000) and press **PMT**. Since we have only payments to deal with, there is nothing to enter as a **FV** . Now solve for the interest rate by pressing the **CPT** key followed by **I/Y**. The calculator screen goes blank for a few seconds and then displays the answer, 12.04%.

You should practice your calculator skills by entering the information from the examples below and calculating the IRR. You may also use Table 3 and the table procedure described above to approximate these IRR values.

- What is the IRR for a $700 investment that promises to provide a series of 10 payments of $100 each? (7.07%)

- What is the IRR for a $10,000 investment that promises to provide a series of 5 payments of $3,000 each? (15.24%)

- What is the IRR for a $20,000 investment that promises to provide a series of 50 payments of $600 each? (1.72%)

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VIII. Alternative Capital Budgeting Techniques – Payback Period and the Average Rate of Return. These two traditional methods of evaluating capital asset investments are still used in practice though they can produce very unreliable results. Each is based upon the decision rule developed above (*namely*, *accept the investment opportunity if the provides a rate of return that is equal to or greater than the hurdle rate*), but they are flawed in the way that they apply the rule. However, they do offer the advantage of simplicity. They are easily understood and easily applied.

    A.   The ***Payback Method*** utilizes the same expected cash flows in its computations as the NPV and IRR methods discussed earlier. It is calculated by determining the number of periods (usually years) required before the original investment is recovered.

B.   For example, if an asset costs $1,000 and promises to provide annual cash returns of $200 over its 10-year useful life, payback occurs in 5 years ($1,000 cost divided by $200 per year). If the asset is expected to return $500 in the first year, $300 the next year, $200 in the third, and $100 per year thereafter, the payback period is 3 years ($500 + $300 + $200 = $1,000).

C.   *Given uniform annual cash inflows*, the payback period can be calculated as:

Payback Period = Cost of Investment ÷ Annual Cash Inflow

1.   If the cash flows are not uniform (as was the case in number 2 above), the formula cannot be applied. Instead, each year's cash flow must be added up, year by year, until payback is achieved. The number of years that are required before the initial cost of the investment is recovered is the payback period.

2.   The presumption in payback analysis is that the earlier payback occurs the more profitable the investment must be. But his is true only if the investment provides uniform annual returns through its useful life and the returns continue to be received well beyond the payback period.

    a.   To illustrate the flaws inherent in the method, consider assets X and Y which each cost $1,000. Asset X promises to return $1,000 in one year, and nothing thereafter. Asset Y promises to return $1,000,000 in two years but nothing in the first year. Asset X's payback period is one year while asset Y requires two years for payback, so according to the payback method asset X should be selected. However, Y is clearly the better investment.

    b.   Had the investments provided uniform returns over equal lives, the payback method would result in the selection of the more profitable investment. For example, if X provided returns of $500 per year for 10 years and Y provided returns of $600 for 10 years; then Y would have the shorter payback period (1.7 years) than X (2 years) and would be selected. In this case Y is the more profitable investment.

Instructor's Lecture Notes:
Another of payback's shortcomings concerns the lack of any direct consideration of the investment's rate of return or the hurdle rate. How can payback be used, then, in applying the capital asset decision rule (i.e., accept only those assets that provide returns equal to or greater than the hurdle rate)?

Companies using the payback method often establish a "minimum required payback" period, based upon the hurdle rate. For example, a company with a hurdle rate of 10%

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knows that 10% (one tenth) must be earned on its capital asset investments. Therefore, if one-tenth of the cost of the investment must be returned each year, it will take 10 years to recover the cost of the investment. Since it is assumed that payback must occur in 10 years or sooner in order to earn a rate of return of at least 10%, 10 years becomes the minimum required payback period.

This process is subject to the same flaws discussed earlier. For example, a $1,000 investment that returns $100 per year for 10 years and then nothing more has a payback period of 10 years, but it is clearly not a profitable investment since profits were not earned at all.

D. The *Average Rate of Return* Method (also called the *Accounting Rate of Return* or *ARR*), is not based on present values. It is a procedure in which the rate of return provided by the asset (i.e., its IRR) is *estimated*. It is calculated as follows:

$$ARR = \frac{\textit{Average Annual Accounting Net Income}}{\textit{(Cost of Investment + Residual Value)/2}}$$

E. Once the ARR is calculated, the same decision rule used with IRR is again applied: if the ARR is greater than or equal to the hurdle rate, the investment should be made. Unfortunately, the method, while simple to apply and easy to understand, merely *estimates* the true rate of return provided by the asset. Often the error in the estimate is large.
1. ARR is *sometimes* (this is not done in the text) calculated by merely dividing the asset's average annual dollar return by its cost. The average annual return is simply the total cash return expected from the asset during its useful life divided by the number of years of useful life.

$$\textit{Simplified ARR} = \frac{\textit{Average Annual Cash Flow}}{\textit{Cost of Investment}}$$

Here's an Example: Asset 1 costs $10,000 and promises to provide cash returns of $4,000 per year for 4 years. The total cash return is $16,000 ($4,000 x 4 years). Dividing by 4 years produces an average cash return of $4,000 per year, and the ARR would be calculated to be 40% per year ($4,000/$10,000). The true IRR is actually 21.9% (given PV = $10,000, N=4, PMT=$4,000, then I = 21.9%).

2. To illustrate one factor that caused this error to occur, consider a different asset. Asset 2 costs $1,000 and has a useful life of one year. At the end of the year, $400 will be earned, and the asset will then be scrapped with no salvage value. Obviously, the asset produces a loss of $600 over its 1-year useful life since $1,000 is invested and only $400 is recovered. However,

the average annual cash return is $400 and the ARR would be determined to be 40% ($400÷$1,000). The actual IRR is a *negative* 60%.

3. The reason this problem occurs is because the ARR method, as calculated above, does not take into account the repayment of the original investment. The annual returns are assumed to represent "pure" cash profits and do not factor in the return of principal. For example, Asset 2 would have to actually produce a year-end payment of $1,400 ($1,000 return of investment plus $400 in cash profit) in order to generate a true 40% rate of return. (The ARR in this case would be 140%!).

4. Conclusion: the ARR as calculated above would only equal the true rate of return if the asset had an unlimited life. In that case, the problem of the return of the original investment would never arise. Otherwise, the ARR as calculated above will consistently overstate the asset's true rate of return, though the longer the asset's useful life the smaller the effect will be.

5. In order to address the problem of return of principal, the ARR method, as it is usually applied (and as it is presented in our text), substitutes average accounting net income in place of average annual cash inflow; and divides this by the average investment cost instead of the total original cost.

$$\textit{Average Rate of Return} \quad = \quad \frac{\textit{Average Annual Accounting Net Income}}{\textit{Average Amount Invested in the Asset}}$$

a. ***Average annual accounting net income*** is just the additional reported accounting net income that results if the asset is acquired. This would be equal to the differential revenues provided by the asset less the differential cash expenses incurred through its use *and its depreciation expense*:

> **Cash Revenues from use of Asset**
> **- Cash Expenses from use of Asset**
> **- Depreciation Expense for Asset**
> **= Accounting Net Income**

Subtracting depreciation provides for the return of the asset's original cost. That is, we allow for $2,500 per year (a total of $10,000) to be taken out of each year's net income, and used to "repay" the company for its investment. In fact, depreciation is sometimes referred to as "cost recovery."

b. The ***average investment cost*** is an approximation of the average amount invested in the asset throughout its life. As we explained above, it is assumed that a portion of the cash returns that are

received annually from the investment represent a return of the asset's cost, and that the remainder is profit. Therefore, the total initial cost of the asset does not remain invested in the asset throughout its life.

1.   If there is no residual value at the end of the asset's life, then the average amount invested begins with the total cost, but then falls to zero when the last cash return is received. For example, suppose that a $1,000 asset investment has a 2-year life. It would be assumed that $500 of this cost is returned with the first annual cash return received from the investment, and that the final $500 is received from the second return. Therefore, on average, only $500 ($1,000 divided by two years) has been invested in the project during its two-year life.

Let's Continue the Example: Recall our earlier example. Asset 1 cost $10,000 and provided net cash returns of $4,000 per year for 4 years. However, if the asset is depreciated using the straight-line method with no residual value, depreciation will be $2,500 per year ($10,000 ÷ 4), and the annual net income will be $1,500 ($4,000 – 2,500). Also, if there is no residual value, the average amount invested in the asset will be $5,000 [($10,000 + 0) ÷2]. Now the ARR would be calculated to be 30% per year ($1,500 ÷ $5,000). This is closer to the true IRR of 21.9%, but it is still very much in error.

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2.   Why do we add the residual value when we calculate the average investment? Suppose the $1,000 asset described in #1 above has a $200 residual value that is received at the end of its 2-year life. Since we do not recover receive $200 of the $1,000 investment at the end of the second year. This $200 portion of the cost remains fully invested during the asset's life. The $800 remainder, however, is received annually throughout the asset's life. Now the average investment is equal to $400 ($800/2) plus the residual amount of $200, or $600.

3.   Therefore, the average investment cost is equal to:

$$\textit{Average Investment Cost} \quad = \quad \frac{\textit{Total Original Cost} - \textit{Residual}}{2} \quad + \textit{Residual}$$

4.   Mathematically, this can be rearranged as follows:

$$\textit{Average Investment Cost} \quad = \quad \frac{\textit{Total Original Cost} - \textit{Residual}}{2} \quad + \textit{Residual}$$

Continued from above

$$= \frac{\text{Total Original Cost}}{2} - \frac{\text{Residual}}{2} + \text{Residual}$$

$$= \frac{\text{Total Original Cost}}{2} + \frac{\text{Residual}}{2}$$

$$= \frac{\text{Total Original Cost} + \text{Residual}}{2}$$

Comprehensive Example – NPV, IRR, Payback and ARR

Elgin Corporation plans to expand operations and is considering two new welders. Welder C costs $10,000, and promises to provide cash revenues of $8,000 per year over its 5-year useful life. It will also generate cash expenses of $4,000 per year. Welder D costs $25,000, and promises to provide cash revenues of $15,000 per year over its 5-year useful life. It will also generate cash expenses of $6,000 per year. Elgin's hurdle rate is 12%. Should either of the welders be purchased?

Step 1 - Identify differential cash flows and accounting net income from each welder:

	Welder C	Welder D
Cash Revenue	$ 8,000	$ 15,000
Cash Expense	(4,000)	(6,000)
Net Cash Flow	$4,000	$ 9,000
Depreciation Expense	(2,000)	(5,000)
Accounting Net Income	$2,000	$ 4,000

Step 2 - Calculate NPV, IRR, Payback and ARR:

NPV:
NPV of Welder C:

Present Value of Cash Flow (n=5,I=12,pmt=4,000)	$14,419
Cost of Asset:	(10,000)
Net Present Value:	$ 4,419
Decision:	Accept

NPV of Welder D:

Present Value of Cash Flows(n=5,I=12,pmt=9,000)	$32,443
Cost of Asset:	(25,000)
Net Present Value:	$ 7,443
Decision:	Accept

IRR:

IRR of Welder C (n=5,pv=10,000,pmt=4,000):	28.7%
Decision:	Accept

IRR of Welder D (n=5,pv=25,000,pmt=9,000):	23.4%
Decision:	Accept

Payback Period:

Payback for Welder C:	$10,000
Cost of Asset:	÷ 4,000
Divide by Annual Cash Flow:	2.5 years
Payback Period	?
Decision:	

Payback for Welder D:	
Cost of Asset:	$25,000
Divide by Annual Cash Flow:	÷ 9,000
Payback Period	2.8 years
Decision:	?

Note that payback does not lend itself to accept/reject decisions unless the company has established a minimum required payback period. At a 12% hurdle rate, the company might establish a required payback of 8.3 years (100%÷12%=8.3%) since it would require that 12% of its investment be returned as profit each year (see the discussion above). If this were the case, then both investments would be accepted. The determination of required payback is not in the text and you are not responsible for its calculation.

Average Rate of Return (ARR):

ARR for Welder C:

Differential Accounting Net Income:	$ 2,000
Divide by Average Cost of Asset [(10,000+0)/2]:	÷ $ 5,000
Average Rate of Return (ARR):	40%
Decision:	Accept

ARR for Welder D:

Differential Accounting Net Income:	$ 4,000
Divide by Average Cost of Asset [(25,000+0)/2]:	÷ $12,500
Average Rate of Return (ARR):	32%
Decision:	Accept

Click the link below to play a video lecture that "walks you through" the comprehensive example above.

Comprehensive Example Presentation

E. ***ARR and Payback compared to NPV and IRR***. In the example above, NPV and IRR both indicated that the two assets should be acquired. NPV and IRR are theoretically sound methods in that they are based on the true rate of return provided by the asset rather than an approximation. Use of the traditional methods introduced here, ARR and the payback period, would also result in the selection of these assets. However, as indicated earlier the methods are unreliable due to error in their approximation of the true rate of return. For example, the ARR is 40% and 32% for C and D, respectively, but their true rates of return, the IRR's, are well below 30%.

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IX.   Capital Rationing and Mutually Exclusive Investments.

A. This topic concerns "either/or" investment decisions. For example, in the illustration above a company was analyzing two welders, C and D. Both were profitable, so the company accepted both. However, if the two welders were being analyzed in order to choose the best one to use to replace a worn out welder, then the company could only select one. Also, if the company is at the end of its capital budget, then funds many not be available to purchase both assets. In either case C and D are *mutually exclusive* investments. The selection of one automatically excludes the other.

B. **IRR**. The safest way for the company to choose the more profitable welder is on the basis of IRR. Since IRR is a measure of the rate of return provided by each of the assets, it is easy to determine which is the most profitable. In the example above, welder C's IRR was greater than welder D's IRR, so C should be selected.

C. **NPV**. The NPV method is difficult to interpret in making mutually exclusive investment decisions. In the example above, welder D's NPV is greater than C's NPV. However, D costs a good deal more than C, so its NPV should be greater. This cost difference introduces a favorable bias in D's NPV value. A way to remove this bias is to look at the present value of the future cash flows produced per dollar invested in each of the assets. This measure is called the ***present value index*** *(also called the profitability index):*

Present value index = PV$_{\text{of cash flows}}$ ÷ Cost of Investment

Applying this measure to welder C and welder D produces the following:

Present value index for Welder C:

Present Value of Cash Flows (n=5,i=12,pmt=4,000):	$14,419
Cost of Asset	÷ 10,000
Present Value Index:	1.4419

Present value index for Welder D:

Present Value of Cash Flows (n=5,i=12,pmt=9,000):	$32,443
Cost of Asset:	÷ 25,000
Present Value Index:	1.2977

Decision: *Select Welder C.*

1. Note that the present value index provides the same result as IRR, the selection of welder C. Even though the present value of C's future cash flows is less than D's, it provides more PV per dollar invested and is, therefore, more profitable. The present value index will usually result in the same decision as IRR because both measure the same thing - the return received per dollar invested.

2. The problem of *ranking* capital asset investments is similar to the problem of mutually exclusive investments. This may occur when several profitable opportunities are available but funds are limited. The approach taken is to *rank* the investments in order of their expected profitability (as measured by IRR or the Present value index), and then select the most profitable first followed by the next most profitable investment and so on, until no more can be taken.

C. **Payback** and **ARR** provide mixed signals in the example above. Since welder C's payback is slightly less than D's, payback would favor C. The ARR is identical for both assets, so no indication is provided. As we know, both of these simplified methods are subject to error and the signals they produce regarding the selection of assets should be viewed with suspicion.

X. Complicating Factors in Capital Budgeting.
A. One of the greatest difficulties in capital budgeting is identifying and quantifying the cash flows that will result from the acquisition of a capital asset. In the new manufacturing environment, characterized by JIT inventory systems and total quality management, it is very difficult to forecast the benefits that will be derived from investment in some capital assets, particularly when the investment involves new technology or contributes in an *intangible* way to maintaining customer satisfaction. For example, an investment may have a negative NPV, but it could be argued that if the investment is made the company will be better positioned to take

advantage of technological improvements in the future, will impress its customers, and will be more flexible in its ability to produce new products as they are developed in the future. However, these benefits cannot be measured in dollar terms and included in the NPV calculation. As a result, many companies are now considering "intangible" factors in addition to the more quantifiable cost and revenue projections when making capital budgeting decisions.

B.   ***Income tax effects*** are important considerations in capital budgeting decisions since they affect cash flows. In the previous examples all cash inflows were "before-tax" amounts. Since the corporation must pay income taxes on taxable income, the amount left after taxes will be less than the before-tax amounts. Suppose a corporation that is subject to a 30% income tax rate earns $1,000 of taxable revenues. After paying taxes of $300, only $700 is left "after taxes." It is these ***after-tax cash flows*** that should be used in analyzing capital expenditures.

C.   ***Unequal project lives.*** When investments have unequal lives, adjustment must be made to prevent distortions caused by timing differences to enter into the capital budgeting analysis. This can be done by assuming that the longer-lived project is disposed of at the time the shorter-lived investment ends. A residual value must be estimated for the long-lived asset and included in computing its NPV and IRR.

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Instructor's Lecture Notes: The following illustration attempts to better explain the meaning of NPV and IRR, and to explore a topic referred to as the "reinvestment rate assumption." Textbooks often point out that *the NPV method implicitly assumes that cash returns received during a project's life are reinvested at the hurdle rate, while the IRR method assumes that they are reinvested at the internal rate of return.* This is true, but it is impossible to appreciate this truth unless it is illustrated through an example.

Assume that a $1,000 project is being analyzed. The project will have a 4-year life, promises to generate cash returns of $200 each year, along with $1,000 at the end of the fourth year. The $1,000 required investment will be financed through owner and creditor contributions to the business that have an overall cost of 10%. Calculating the NPV for the project (with n = 4, i = 10%, pmt = $200, and FV = $1,000) results in a positive $316.99. The IRR (with n = 4, pv = $1,000, pmt = $200, and FV = $1,000) is 20%. Therefore, the project should be accepted.

Reinvestment Rate Assumptions. Since the company must "pay" a 10% rate of return to the providers of the $1,000 of capital, a rate of return of at least 10% must be earned from the project if it is to be undertaken. Therefore, the hurdle rate is 10%, and $100 must be distributed to the providers of capital ($1,000 x 10% = $100) each year in order to give them their required 10% return. At the end of the project's life it will then be necessary to return the $1,000 to them, so that they will have invested $1,000 in the project, earned $100 per year on it, and then received their $1,000 investment back. This gives them their

required 10% rate of return. It follows, then, that anything received from the project over and above those returns belongs to the company and its owner. These "excess returns" belong to the business owner, and have the effect of increasing the owner's wealth.

NPV. In the illustration below, the project's cash returns are mapped out on the time line, and the required payments to the providers of the capital are also drawn in. The amount left over represents "excess returns" that accrue to the owner. If we assume they are reinvested at the hurdle rate of 10%, they will grow, year by year, by 10%. The arrows in the table below indicate the growth of the excess returns at 10%.

Time (Years)	0	1	2	3	4	
Cash Flows	-1,000	+200	+200	+200	+ 200 +1,000 +1,200	
Payment to Investors Excess, Reinvested @10% Total Payment to Investors Excess, Reinvested @10% Total Payment to Investors Excess, Reinvested @10% Payment to Investors Return of Capital Excess Left for Owner		-100 +100 ⟶	110 310 -100 210 ⟶	231 431 -100 331 ⟶	364.10 1,564.10 -100.00 -1,000.00 464.10	

As shown above, if the "excess" cash returns are reinvested at the hurdle rate (10%), the owner has $464.10 left over after giving the providers of the $1,000 of capital a 10% rate of return on their investment in the business, and after returning their invested capital to them. This increase in wealth, however, is a "time 4" future value amount. The present value of this amount, discounting at 10% per year, is $316.99, which is equal to our NPV. This illustrates the fact that the NPV measures the increase in wealth that accrues to the business after the capital is obtained and invested in the project, required returns (at 10%) are given to the providers of the capital, and the capital is returned to the investors at the end of the project's life. However, **the $464.10 increase in wealth can only be obtained if the cash flows received during the project's life are reinvested at exactly 10% per year**. Conclusion:

> *NPV is a direct measure of the amount by which the business owner's wealth increases if the project being analyzed is undertaken, assuming that all the intermediate excess cash returns retained in the business during the project's life are reinvested at the hurdle rate.*

IRR. In examining the reinvestment rate assumption inherent in IRR, we need to take a slightly different approach. When we say that the project's IRR is 20%, we are saying that 20% will be earned, overall, on the $1,000 investment through the project's 4-year life. In order for this to be the case, both the "excess" cash returns *as well as the payments to the providers of capital* must be reinvested at 20%. The red arrows in the illustration below represent the reinvestment of the cash returns given to the providers of capital at the 20% rate. The dark arrows represent the reinvestment of the excess returns at the 20% IRR.

Time (Years)	0	1	2	3	4	
Cash Flows	-1,000	+200	+200	+200	+ 200 +1,000 +1,200	
Payment to Investors Excess, Reinvested @20% Total		-100 +100	120 320		$ 172.80	
Payment to Investors Excess, Reinvested @20% Total			-100 220	264 464	$ 144.00	
Payment to Investors Excess, Reinvested @20% Total Accumulated				-100 364	$ 120.00 436.80 $2,073.60	

As is shown above, the reinvestment of "excess" cash returns **and** the returns paid to the providers of capital at the internal rate of return (20%) results in a total accumulation of cash over the project's life of $2,073.60. Since the initial investment is $1,000 and a future value of $2,073.60 is accumulated in 4 years, the rate of return earned on the investment is 20% ($1,000 grows to $2,073.60 over 4 years at a 20% rate of interest). Note that the $2,073.60 can only be achieved if the total cash returns received from the investment ($200) is reinvested at the 20% IRR. Any other reinvestment rate will not produce the $2,073.60 future value and will result in an actual rate of return of something other than 20% per year. Therefore, **the IRR computed for an investment can only be achieved if the intermediate cash flows from the investment are reinvested at the internal rate of return.** Conclusion:

The IRR represents the rate of return that will be earned by undertaking the investment opportunity, assuming that all the intermediate cash flows received from the investment are reinvested at the internal rate of return.

<​back>

-END-

Supplement A
Financial Calculator Instructions (TI Business Analyst)

Initial Setup

1. Set Decimal Places to six. Press 2nd CLR TVM [6] 2nd Set Enter 2nd Quit .

2. Choose Algebraic Operating System (AOS). Press 2nd Format ↑ ↑ ↑ ↑ 2nd Set Enter 2nd Quit .

3. Set Payments per Year (P/Y) to one. Press 2nd P/Y [1] Enter 2nd Quit .

4. Set payments to end-of-year: Press 2nd BGN. If display reads AEND@, the payments are already set to end-of-year. Exit by pressing 2nd Quit . If display reads ABGN,@ press 2nd Set , so that display reads AEND@. Then exit by pressing 2nd Quit .

Note: Your calculator requires that you input the present value amounts as negative values (they represent cash outflows - payments into the investment), and future values or payments as positive values (they represent cash inflows - returns from the investment). Before doing any compound interest problem, clear your calculator by pressing 2nd CLR TVM . Values are stored in the calculator, and the numbers you entered to work earlier problems can affect your results unless you clear the calculator.

Calculations with Single Amounts
In a single-amount problem, we are given three of the four possible values (N, I/Y, PV, and FV) and we are asked to solve for the one that is unknown.

Example: Assume N = 10, I/Y = 6%, PV = $ -100, and FV = $179.08. Use the following keystrokes to solve for the missing value.

First, clear calculator by pressing 2nd CLR TVM . Then, to solve for:

- Future Value: Input 10 N , 6 I/Y , and 1 +/- PV . Press CPT FV .

- Present Value: Input 10 N , 6 I/Y , and 179.08 FV . Press CPT PV .

- Interest Rate: Input 10 N , 1 +/- PV , and 179.08 FV . Press CPT I/Y .

- Number of Periods: Input 6 I/Y , 1 +/- PV , 179.08 FV . Press CPT N .

Calculations with Annuities (a series of equal payments)

In a present value annuity problem, we are given three of four possible values (N, I/Y, PMT, and PV) and are asked to solve for the unknown. In this module, you may be given the Number of Payments (N), the Interest Rate (I/Y), and the Payment (PMT) of an investment, and asked to solve for the Present value (PV). You may also be given all the other inputs and asked to solve for the Interest Rate (I/Y). Remember to clear the calculator between problems, using $2nd$ | CLR TVM |.

Example: N = 5, I/Y = 8%, PMT = $ -100, and PV = $ 399.27. To calculate the:

- Present Value: Input 5 | N |, 8 | I/Y | , and 100 | +/- | | PMT |. Press | CPT | | PV |.

- Payment: Input 5 | N |, 8 | I/Y | , and 399.27 | PV |. Press | CPT | | PMT |.

- Interest Rate: Input 5 | N |, 1 | +/- | | PMT |, 399.27 | PV |. Press | CPT | | I/Y |. This is the Internal Rate of Return of the annuity.

- Number of Payments: Input 8 | I/Y |, 1 | +/- | | PMT |, and 399.27 | PV |. Press | CPT | | N |.

Calculations with A Mixed Stream of Payments

These problems give you series of payments, but since they are not all the same amount they do not represent an annuity. We must calculate the present values or future values of each separately, and then add them all together. We may do this, using the methods shown above for single amounts, though it can be a tedious process if there are very many cash flows involved.

The calculator does have functions that will do all this for us, though using them can be a little intimidating at first. We will need to use the following keys:

| Enter | | ↓ | | ↑ | | C/F | | NPV | | IRR |

The | C/F | key is used to enter the cash flows that are received each period during the investment's life. CF_0 is the time zero cash flow (usually the purchase cost of an investment, entered as a negative amount), CF_1 is the cash flow at the end of the first period, CF_2 is the cash flow at the end of the second period etc. Enter cash inflows as positive amounts and outflows as negative amounts. The NPV (Net Present Value) is the present value of the cash flow stream, including CF0, at the given rate of interest. The IRR (internal rate of return) is the interest rate at which the NPV is zero.

Assume the cash flows consist of $ -100, $120, and $120 at times zero, one, and two. The interest rate is 7%. Here are the instructions for inputting this cash flow stream into your calculator and calculating the NPV and IRR.

Clear the calculator, this time by pressing | C/F | | 2nd | | CLR Work |.

Enter all cash flows by pressing C/F 8 +/- Enter, ↓ 4 Enter, ↓ 5 Enter.

Calculate the NPV by pressing NPV 7 Enter ↓ CPT.

Calculate the IRR by pressing IRR CPT.

Bond Calculations

Though bonds are not covered in our Managerial Accounting course, present value calculations are required when working with them and the following section is included for the sake of completeness. In a bond problem, we are given four of five possible values (N, I/Y, PMT, PV, FV), and we are asked to solve for the unknown. For example, you may be given the number of interest payments (N), the coupon payment (PMT), and the bond price (PV), and par value (FV) and ask to solve for the interest rate (yield to maturity).

Suppose you are asked to solve for the yield to maturity on a five-year bond with an $8 annual coupon payment, a $100 par value, and a market price of $100.

- Yield to Maturity: Input 5 N, 8 PMT, -100 PV, and 100 FV. Press CPT I/Y.

If you are given an 8% annual interest rate, then the price can be found as follows:

- Price: Input 5 N, 8 I/Y, 8 PMT, and 100 FV. Press CPT PV to calculate the bond's market price.

If interest is paid semiannually (10 semiannual periods over 5 years):

- Yield to Maturity: Input 10 N, 4 PMT, -100 PV, and 100 FV. Press CPT I/Y. Result is rate over six months, so double the number to obtain the annual rate.

If you are given an 8% interest rate, compounded semiannually, then the price can be found as follows:

- Price: Input 5 N, 4 I/Y, 4 PMT, and 100 FV. Press CPT PV to calculate the bond's market price.

-End-