

# ***Online Course Manual***

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## **Module 4**

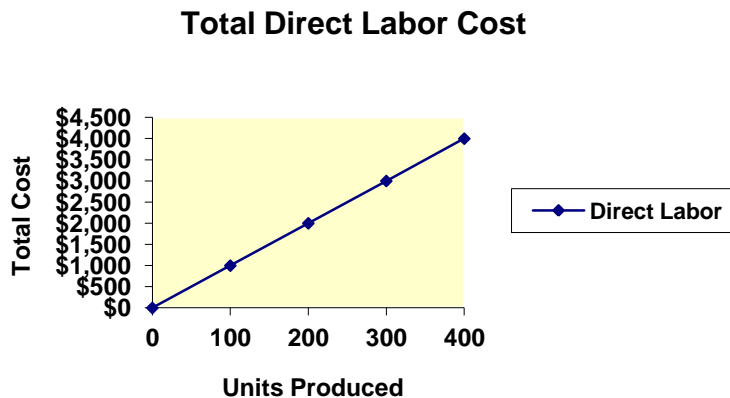
<p style="text-align: center;"><b>Table of Contents</b></p> <ul style="list-style-type: none"><li>I. <a href="#"><u>Cost Behavior</u></a></li><li>II. <a href="#"><u>Contribution Margin</u></a></li><li>III. <a href="#"><u>Cost-Volume-Profit Analysis (CVP)</u></a></li><li>IV. <a href="#"><u>Planning and Decision-Making Applications</u></a></li><li>V. <a href="#"><u>Multiple Products, Sales Mix and CVP</u></a></li><li>VI. <a href="#"><u>Assumptions In CVP Analysis</u></a></li></ul>	<p><b>Instructions:</b></p> <p>Click on any of the underlined titles in the table of contents to be directed to that section of the module. Click on the &lt;back&gt; symbol to return to the table of contents. Click on underlined words to be linked to the section.</p>
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## Module 4 Summary

**I. Cost behavior** refers to the way that costs change as production levels change. As we know from the previous module, cost drivers are factors that cause cost to be incurred. In this module, the cost driver we focus on is simply the level of production. We will examine the way that costs change as the quantity of units manufactured increases. Several terms are used to describe this behavior:

A. **True Variable Costs** are costs that vary *directly and in proportion* to changes in activity. While costs that fit this definition exactly probably do not exist, direct materials, direct labor, and some overhead costs come close to meeting the technical definition of a variable cost and are treated as such.

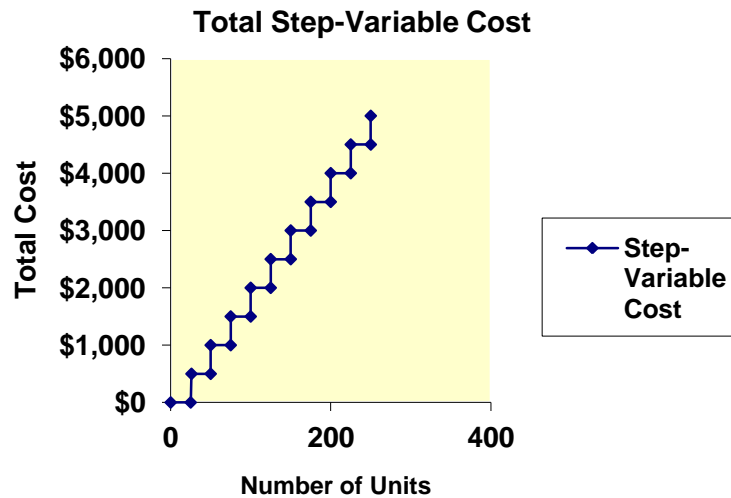
1. The term "direct" means that the cost increases as activity (i.e., the quantity produced) increases. The phrase "in proportion" means that the cost changes by the *same amount* each time the activity level is increased by one more unit of production. This means that the **variable cost per unit is a constant amount**; and that the total variable cost, when plotted on graph paper against production in units, forms a straight up-sloping line. In these cases it is said that there is a *linear relationship* between the cost and the activity level. To illustrate, assume that it takes a worker one hour to produce a unit. If the worker is paid \$10 per hour, then the *total* direct labor cost of manufacturing units is a variable cost that, if charted, would look like so:



2. Note that the variable cost *per unit* remains constant at \$10, no matter how many units are produced. This constant rate of change in the cost is characteristic of all variable costs.
3. Other variable costs that would behave in the same way are *direct materials* and *variable overhead* costs, such as utilities, indirect labor and indirect materials.

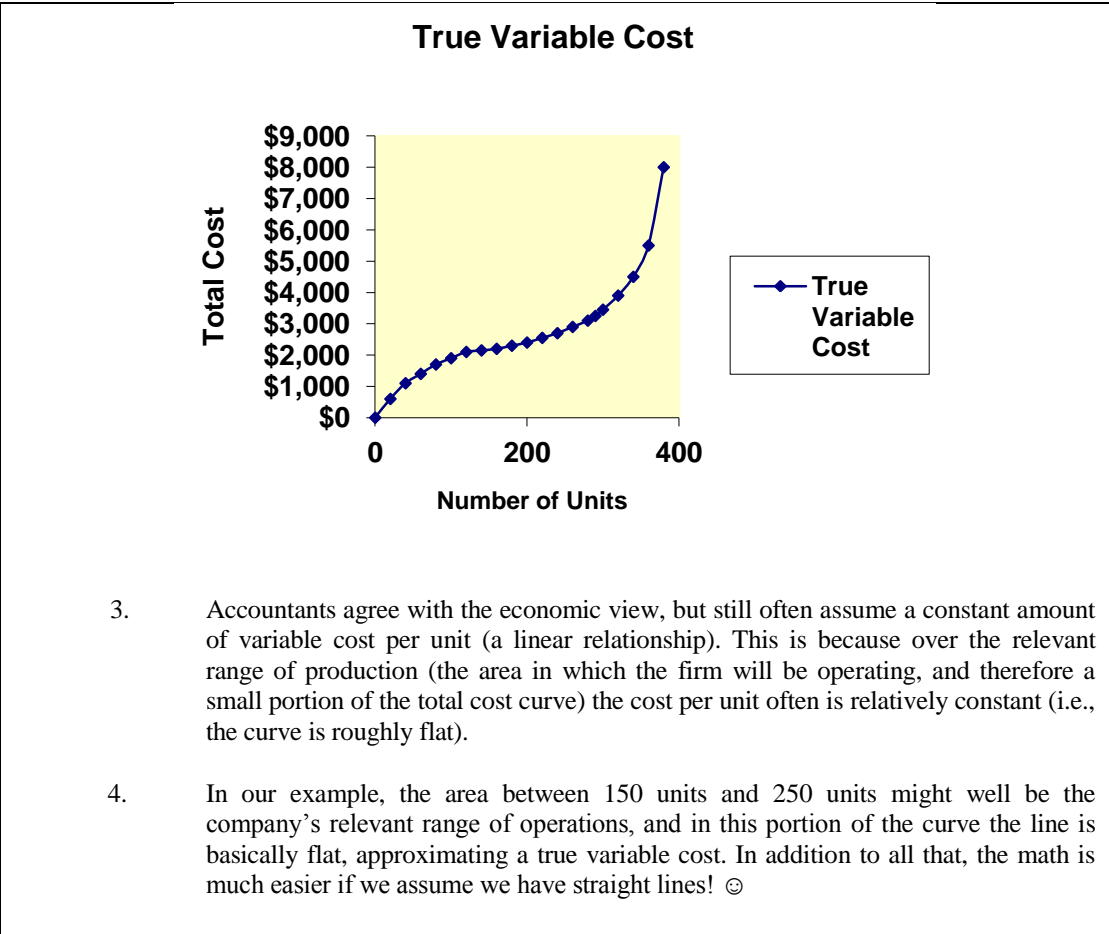
*Instructor's Lecture Notes.* This discussion is supplemental information that you will not be tested over directly. However, you are encouraged to read it over because it will help you fully understand the module's content.

**Step-Variable Costs** are costs that do change with production, but not "smoothly." That is, there is a "lag" as more units are produced before the cost "steps up." If the lag is very short, then the cost behaves very much like a true variable cost and it can be accounted for as such. A step-variable cost would graph as follows:



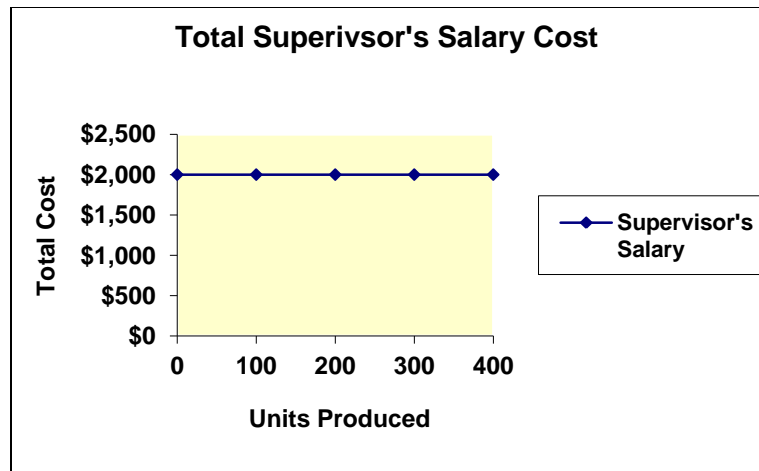
The "*economist's view*" holds that variable cost per unit decreases at first as efficiencies (called "economies of scale") are realized, and then at some point begins to increase as inefficiencies are encountered. They would say that the variable cost curve is NOT a straight line (a *linear* relationship), but is instead "curved" (a *curvilinear* relationship).

1. For example, suppose Farmer Brown grows tomatoes on his 1,000-acre tomato field with only one hired hand. Because much of the crop cannot be attended to with only one worker, the number of tomatoes produced per direct labor hour will be smaller than it would be with two helpers. Therefore, the direct labor cost per tomato produced will fall as more workers are added, but only to a point. Eventually there will be too many workers. As more are added, they will get in each other's way, step on the tomatoes, and no longer produce an increasing number of tomatoes per direct labor hour. At this point the labor cost per tomato produced begins to rise.
2. This example illustrates the fact that the labor cost line plotted on graph paper is not a straight line. Instead, it forms an "S," curving downward at first as production increases per labor hour used and efficiencies are realized, and then turning upward as the inefficiencies are encountered. When graphed, a "true" variable cost curve would appear so shown below:



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- B. **Fixed costs** are costs that do not change as activity changes.
1. If total fixed costs are graphed against production in units they form another straight line, but a flat one instead of one that slopes upward since they are constant in amount no matter what the level of production. Again, as was the case with variable costs, there are probably no costs that are truly fixed, but examples of costs that tend to remain fixed include depreciation, property taxes, supervisor salaries, etc.
  2. A graph of a fixed cost, Supervisor Salaries, would appear as shown on the page below.
  3. Over the “long run,” there are probably no costs that remain fixed, since, at some point, additional plant assets may be purchased (increasing depreciation and property taxes), new supervisors hired (increasing salaries expense), and so on in order to support a higher level of production.



4. This means that the costs we will call “fixed” are actually costs that will eventually “step up” to a new, higher level; but *large* changes in activity are required before the “step” occurs. However, in the “short run” and over the *relevant range* of operations these costs will not change and the problem of steps can be ignored.
5. As JIT systems with their extensive use of automated equipment become the norm in the modern manufacturing environment, fixed costs have come to represent a much larger portion of total costs than they previously did.

Lecture Notes, Continued. Fixed costs place restrictions on companies, since they are always “there” and must be covered by revenues if the company is to make a profit. In planning operations, the manager must distinguish between *committed* and *discretionary* fixed costs.

**Committed fixed costs** are long-term, continuing costs that cannot be reduced without impacting on the company’s ability to carry on its operations. Examples are depreciation of plant facilities, insurance, property taxes and executive salaries.

**Discretionary fixed costs** are short-term costs that can be eliminated or reduced without affecting the company’s ability to carry on its present operations. Research and Development Department costs are often viewed as discretionary in nature.

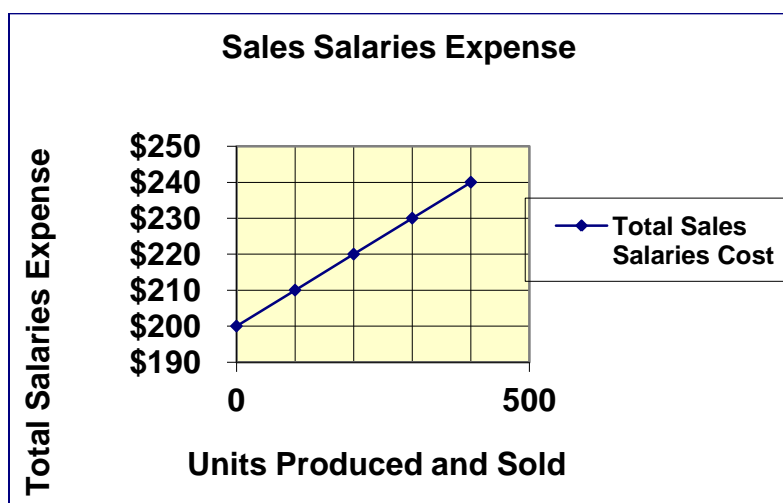
Because of the inflexibility that committed fixed costs produce, managers try to find ways to reduce them. If it is possible to structure business operations in ways that provide for more flexibility – that is, that reduce committed fixed costs and replace them with discretionary fixed costs – then managers are quick to do so. For example, equipment might be rented instead of purchased, “temps” from personnel services companies might be used instead of full-time employees, parts might be purchased from outside suppliers rather than manufactured in-house. All these actions make it easier for the company to down-size production, switch product lines, or enter into new types of business operations without severe and costly disruptions.

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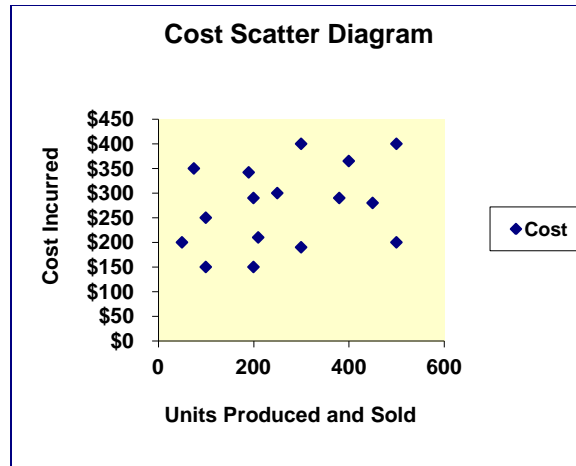
- C. **Mixed costs** (also called *semivariable costs*) have elements of both fixed and variable costs. An example would be a salesman's \$200 per week

base salary that is enhanced by a 10% commission based upon sales. The base salary is a fixed cost and the commissions are variable costs, but if only one expense account, Sales Salaries Expense, is maintained in the accounting system, then the account balance would have to be viewed as a “mixed cost.”

1. We may analyze mixed costs by separating the fixed cost component from the variable cost component and treating the two as separate entities. For example, the salesman's salary could be divided into a fixed cost of \$200 and a variable cost per unit that would be equal to 10% of the selling price of the unit.
2. Cost behavior may be analyzed visually (using a *scatter diagram*) to identify and quantify fixed and variable components. The cost in question is analyzed by plotting the total cost incurred during several previous periods against the number of units produced in each period. A line is fitted to the points, and the fixed and variable costs per unit are identified. The salesman's salary would appear as follows:



As can be seen, the \$200 fixed portion of the total expense is clearly discernable, as is the commission. From a visual inspection of the graph, it appears that the commission (the variable portion of the total cost) amounts to 10¢ per unit – note that the salary increases by \$10 every time the number of units sold increases by 100. This “visual fit” works very well in this case, since all the points lie exactly on a straight line, and the results are accurate. However, if the points do NOT all lie on a straight line, interpretation depends upon the “eye of the beholder” and is likely to be much in error. For example, what line would you fit to the scatter diagram below? ☺



3. A mathematical way to determine the variable and fixed components of a mixed cost is to use *the high-low method*.
- The *high-low method* is so named because it utilizes the highest point (largest number of units and largest cost) and the lowest point (smallest quantity and cost) on the graph to determine the variable and fixed costs.
  - The total cost is equal to the fixed cost portion of the cost *plus* the total variable cost. The total variable cost is equal to the variable cost per unit times the number of units. These facts allow us to use the high and the low points to calculate the fixed costs and the variable cost per unit:

- First, subtract the total cost at the low point from the total cost at the high point to determine the change in cost that occurred between the two production levels:

$$C_{high} - C_{low} = Change_{cost}$$

- Next subtract the number of units produced at the low point from the number produced at the high point. This is the change in production that occurred between the two points:

$$U_{high} - U_{low} = Change_{units}$$

- Now divide the change in cost by the number of units that were produced between the two points. This is the increase in cost that occurred with each additional unit produced, or, by definition, the variable cost per unit:

$$Change_{cost} \div Change_{units} = VC_{per\ unit}$$

4. Next choose either the high or the low point, and calculate the total variable cost for that point. This will be equal to the number of units produced multiplied by the variable cost per unit that was calculated in step 3:

$$\text{Units} \times \text{VC}_{\text{per unit}} = \text{Total Variable Cost}$$

5. To determine total fixed cost, simply subtract the total variable cost calculated in step 4 from the total cost incurred at the point chosen.

$$\text{Total Cost} - \text{Total Variable Cost} = \text{Fixed Cost}$$

- c. Applying these steps to the sales salary example results in the following:
  1. The low point is \$200 total cost at zero units produced and sold; the high point is \$240 at 400 units produced and sold.
  2. Subtracting \$200 from \$240 yields \$40, the change in total cost. Subtracting 0 units from 400 produces 400 units, the change in production.
  3. Dividing \$40 by 400 units produces 10¢, the variable cost per unit.
  4. Choosing the high point, we determine \$40 to be the total variable cost (10¢ per unit times 400 units). Therefore, fixed costs are \$200 (the \$240 actual total cost minus the \$40 variable cost).
- d. Note that this method will only produce accurate results if these two points are representative of the relationship between cost and activity. In the sales salaries example, all of the points lay on a straight line so the high and low points produced perfectly accurate variable and fixed cost figures. Had the points been more scattered so that a single straight line was not formed, the method might have produced very inaccurate results.
- e. As was mentioned above, the scatter diagram/visual fit method and the high-low method are both subject to error. Another method, the *least-squares regression method*, does a much better job of fitting an accurate line to the cost points. The least-squares regression method gives us a mathematically precise determination of the variable and



fixed costs, and it is very easy to apply using a calculator with statistical functions or a computer. Unlike the high-low method, it considers every point that is plotted on the graph and it is not subject to errors due to visual estimation as the scatter graph method is. Least-squares regression, using Excel, is illustrated in the supplement at the end of this module.

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## II. Contribution Margin

- A. Because it helps managers separate relevant information from irrelevant information, **contribution margin** is very useful in management decision-making. *Contribution margin* may be expressed on a per-unit basis or as a total amount.

$$\text{Contribution Margin}_{\text{per unit}} = \text{Selling Price}_{\text{per unit}} - \text{Variable Cost}_{\text{per unit}}$$

or

$$\text{Contribution Margin}_{\text{total}} = \text{Net Sales} - \text{Total Variable Costs}$$

- B. A **contribution format income statement** is often prepared for management decision-making purposes. It is similar to the income statements dealt with previously, except that all the variable costs, including variable selling and administrative costs, are grouped together and subtracted from sales. This results in the presentation of total contribution margin on the face of the statement. As a final step, the fixed costs (fixed factory overhead and fixed operating expenses) are subtracted from contribution margin to determine net income.
- a. In the contribution margin income statement below, 50,000 units were produced and sold for \$5 each. The variable cost of manufacturing them (direct labor, direct materials and variable overhead) is \$2 per unit. Fixed manufacturing costs totaled \$100,000. There are \$10,000 of operating expenses, composed of \$5,000 of fixed operating costs and \$5,000 of variable operating costs (\$.05 per unit). The contribution margin income statement appears as follows:

<i>Contribution Margin Income Statement</i>		
Sales (50,000x\$5)		\$250,000
Variable Manufacturing Costs (50,000x\$2)	\$100,000	
Variable Operating Exp.'s (50,000x\$.05)	<u>2,500</u>	
Cost of Goods Sold (50,000x\$2.05)		<u>102,500</u>
Contribution Margin		\$147,500
Fixed Factory overhead Costs	\$100,000	
Fixed Operating Expenses	<u>5,000</u>	<u>\$105,000</u>
Net Income		\$ 42,500

*Note: The total contribution margin, as shown above, is \$147,500. The contribution margin per unit is \$2.95 (\$5 per unit selling price minus \$2.05 in variable costs per unit).*

**III. Cost-Volume-Profit Analysis (CVP)** is sometimes called *break-even analysis* or “profit planning.” The technique applies the concepts outlined previously regarding the way costs behave as the volume of production changes (our discussion about *variable* and *fixed costs*) to many business decisions.

A. **The meaning of contribution margin.** Recall that variable costs increase with each unit produced and sold, just as sales revenue does. Since the company is sure to set the selling price above the variable costs of production, the manufacture and sale of each individual unit creates a gain (economists refer to this gain as *margin*) equal to the difference between its selling price and its variable cost. The greater the number of units produced and sold, the greater the accumulated contribution margin from the sales.

1. However, accumulating margin from the sale of units does not mean the company has a “bottom-line” profit, since the fixed costs have not yet been subtracted. Before profits can be made, enough units will have to be sold to accumulate total contribution margin equal to the fixed costs.
2. Therefore, *before a profit can be made, the company must first produce and sell enough units, each of which “contributes” its individual contribution margin, to cover all of the fixed costs. Once this has been done, the contribution margin from the sale of any additional units “contributes” to bottom-line profits.*

B. The **break-even point** is the level of sales at which the company finally covers its fixed costs with total contribution margin, neither earning a profit nor incurring a loss but just *breaking even*. As indicated above, once enough units are produced and sold to generate total “contribution” that is sufficient to provide for the fixed costs, the company breaks even. Therefore, *to calculate the break-even point it is only necessary to divide the fixed costs by the contribution margin per unit:*

$$BE_{units} = Total\ Fixed\ Costs \div Contribution\ Margin\ per\ Unit$$

or:

$$BEP_{in\ Units} = \frac{FC}{CM_{per\ unit}}$$

*Here’s an Example!* Tommy began a lawn mowing business this summer, charging \$50 per acre of lawn. His variable costs (gasoline, oil, and maintenance) are estimated to be \$30 per acre. The contribution margin is, therefore, \$20 per acre (\$50 - \$30 = \$20). Fixed costs for the summer (depreciation and insurance) will be \$2,000. How many acres of lawn must Tommy mow before he “breaks even” and begins to earn a profit? We are asking, “how many acres, each of which contributes \$20 toward the fixed costs of \$2,000, must Tommy mow in order to cover the fixed costs?”

Answer:  $\$2,000 \text{ FC} \div \$20 \text{ CM}_{\text{per unit}} = \underline{100 \text{ acres}}$ .

Here is the proof that this answer is correct:

$$\begin{aligned} \text{Profit} &= \text{Total Sales} - \text{T. Variable Costs} - \text{T. Fixed Costs} \\ &= (100 \text{ acres} \times \$50) - (100 \text{ acres} \times \$30) - \$2,000 \\ &= \$5,000 - \$3,000 - \$2,000 = \underline{\$0} \end{aligned}$$

Tommy breaks even when 100 acres have been mowed. At any level of activity below 100 acres, losses are incurred since not all the fixed costs will be covered. At any level of activity above 100 acres, profits are earned since the fixed costs are all covered after 100 acres are mowed.



Click the link below to play a video that illustrates and discusses this example and the other illustrations shown in the sections below.

[Link to CVP Examples](#)

- C. Sometimes the *break-even point is expressed in dollars of sales* rather than units of sales. The break-even point in sales could be calculated very easily as follows:

$$BE_{\text{sales}} = BE_{\text{units}} \times \text{Selling Price per Unit}$$

In the example above, Tommy's break-even point in sales dollars is:

$$BE_{\text{sales}} = 100 \text{ acres} \times \$50 = \$5,000.$$

1. This approach will always work, but it requires calculation of the  $BE_{\text{units}}$  before the  $BE_{\text{sales}}$  can be determined. Another way of solving for the  $BE_{\text{sales}}$  involves the use of the *contribution margin ratio*. The CMR is calculated as follows:

***CMR = Contribution Margin per unit  $\div$  Selling Price per unit:***

$$CMR = \frac{CM_{\text{unit}}}{SP_{\text{unit}}}$$

2. The interpretation of the contribution margin ratio is very straightforward. It tells us the percentage of the unit's selling price that is contribution. For example, given the \$50 selling price in the example above and a contribution margin of \$20 per acre, Tommy's CMR is .40 ( $\$20 \div \$50$ ), or 40%. Therefore, *40 cents of every dollar of sales that Tommy makes is contribution*. The other 60 cents has been "lost" to variable costs.
3. This fact allows us to apply the CMR in the same way we applied the  $CM_{\text{per unit}}$  in calculating break-even points earlier. Now we ask, "how many *dollars* (instead of *units*) of sales must be made, each of which contributes 40 cents toward covering the fixed

costs, before they are all covered?” This would simply be equal to the total fixed costs divided by the CMR:

$$BEP_{sales} = \frac{FC}{CMR}$$

We know that Tommy’s CMR is .40 (calculated above). Suppose the fixed costs are \$2,000. The break-even point in *sales dollars* is:

$$BE_{sales} = \$2,000 FC \div .40 CMR = \underline{\$5,000}.$$

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**IV. Other Planning and Decision-Making Applications** stem from the concepts applied above.

- A. ***Predicting profits and losses*** for a given level of activity is a common ***budgeting*** application. Since contribution margin tells us how much each units “contributes” toward covering the fixed costs and then toward bottom-line profit, it is an easy matter to determine what the profit (or loss) will be at any forecasted level of production and sales.
1. Multiply the given number of units expected to be sold by the  $CM_{unit}$  to calculate total contribution.
  2. Subtract the fixed costs from the total contribution margin. If the difference is positive, it represents profit earned at the given level of activity. If negative, it represents an anticipated loss.

***Continuing the Example...*** Suppose Tommy estimates he will be able to mow 500 acres during the summer. Since the breakeven point is only 100 acres, Tommy knows he will earn a profit at that level of activity. How much profit?

Answer:

1. 500 acres x \$20  $CM_{unit}$  = \$10,000 total contribution.
2. \$10,000  $CM_{total}$  - \$2,000 Fixed Costs = \$8,000 profit.

- B. ***Calculating the production level that is required to attain a target profit amount*** (*profit planning*) utilizes the same approach we used earlier to determine the break-even point. Now, however, we are asking “how many units must be produced and sold in order to (1) cover the fixed costs *and* (2) produce some target profit figure above and beyond the fixed costs?” The most understandable way to obtain an answer would be to first calculate the break-even point in units and then calculate how many more units would have to be produced *beyond* break-even in order to provide the profit.

*Our Example, Continued.* Assume that Tommy is not satisfied with an expected profit of \$8,000. He wants to earn \$10,000. How many acres of lawn must be mowed in order to provide the target profit of \$10,000?

1. Tommy knows that break-even occurs at 100 acres.
2. The additional acres beyond break-even that must be mowed to generate an additional \$10,000 in contribution are:

$$\$10,000 \text{ Target Profit} \div \$20 \text{ CM}_{\text{per unit}} = \underline{500 \text{ acres.}}$$

3. Therefore, Tommy must mow 100 acres to break even and 500 more acres to earn the target profit, or a total of 600 acres in order to earn \$10,000.

Note that Tommy could have used another way to calculate the answer. Since he knew that he would earn \$8,000 by mowing 500 acres, he could have asked “how many acres beyond 500 must I mow to make an additional \$2,000 in profit?” The answer would have been 100 acres ( $\$2,000 \div \$20 \text{ CM}_{\text{per unit}}$ ), for a total of 600 (500 + 100 acres).

2. This process can be easily combined and made one step instead of two.

$$\text{Required Sales}_{\text{in Units}} = \text{BE}_{\text{in Units}} + (\text{Target Profit} \div \text{CM}_{\text{per unit}})$$

or, in symbols:

$$\text{Required Sales}_{\text{in Units}} = \frac{FC}{\text{CM}_{\text{per unit}}} + \frac{\text{Target Profit}}{\text{CM}_{\text{per unit}}} = \frac{(FC + \text{Target Profit})}{\text{CM}_{\text{per unit}}}$$

3. Note that the contribution margin ratio could also be applied here:

$$\text{Required Sales}_{\text{in \$}} = \frac{FC}{\text{CMR}} + \frac{\text{Target Profit}}{\text{CMR}} = \frac{(FC + \text{Target Profit})}{\text{CMR}}$$

C. *Determining required selling price or variable cost.* We can now calculate required sales levels, but what if sales quantities are fixed? If the number of units sold cannot be increased, it may still be possible to reach the target income by increasing selling price per unit. The selling price required to produce a target profit can be determined as follows:

1. We know that the general formula for the required sales level for a target profit amount is:

$$\text{Required Sales}_{\text{in Units}} = \frac{(FC + \text{Target Profit})}{\text{CM}_{\text{per unit}}}$$

2. Since the sales level in units is fixed, we could rewrite the formula as:

$$\text{Fixed Sales}_{\text{in Units}} = \frac{(FC + \text{Target Profit})}{\text{CM}_{\text{per unit}}}$$

3. Then, since the unit sales, fixed costs, and target profit are known, we could solve for the necessary contribution margin per unit as follows:

$$\text{Required } CM_{\text{per unit}} = \frac{(FC + \text{Target Profit})}{\text{Fixed Sales}_{\text{in Units}}}$$

4. When we have solved for the contribution margin per unit, the selling price per unit could be found as follows:

Since  $CM_{\text{per unit}} = SP_{\text{per unit}} - VC_{\text{per unit}}$  then

$$\text{Required } SP_{\text{per unit}} = \text{Required } CM_{\text{per unit}} + VC_{\text{per unit}}$$

5. If selling price cannot be increased in order to produce the required contribution margin, we might be able to reach it by reducing the variable costs per unit. How much would the variable cost per unit have to be in order to produce the required contribution margin?

Since  $CM_{\text{per unit}} = SP_{\text{per unit}} - VC_{\text{per unit}}$

$$\text{Required } VC_{\text{per unit}} = SP_{\text{per unit}} - \text{Required } CM_{\text{per unit}}$$

*Our Example, Continued.* Suppose Tommy determines that he cannot mow 600 acres during the summer, but believes he can mow 500 acres. Given a fixed level of production of 500 acres, what price would he need to charge in order to earn the desired \$10,000 profit?

- Tommy knows that fixed costs are \$2,000. If the number of acres mowed are fixed at 500, then:
- $CM_{\text{per unit}} = (\$2,000 \text{ FC} + \$10,000 \text{ target profit}) / 500 \text{ acres} = \$24 \text{ per acre.}$
- $SP = \$24 \text{ CM} + \$30 \text{ VC} = \$54 \text{ per acre.}$

If Tommy decides he cannot charge this much and still make sales, he might think about reducing the variable costs to boost profits. If the selling price is kept at the original \$50 per acre, and the required contribution margin per unit is \$24, then the necessary variable cost amount is:

- $VC = \$50 \text{ SP} - \$24 \text{ CM} = \$26 \text{ per unit.}$

It may or may not be possible to reduce the variable costs by \$4 per acre, and it may not be possible to raise the selling price to \$54 per acre, but it may be possible to combine these two actions, reducing the variable costs by a bit and increasing selling price a little. Therefore, Tommy might now be able to figure out a way to reach his \$10,000 profit goal, but he will not be able to do it without an understanding of the CVP tools. They are needed in order to make effective management decisions and reach the profit goal.

- D. **Managing the Cost Structure.** This application of C-V-P analysis involves determining the effect on profits of anticipated changes in variable cost per unit, or in total fixed costs. Since profit is equal to total selling price minus total variable costs and fixed costs, it is easy to project the effect upon profits that will result from these changes.

*Here's another Example!* Suppose Tommy is considering altering his **cost structure** (the relative mix of fixed and variable cost) in order to increase profits. He is thinking of purchasing a more expensive but more efficient mower. If he does, his fixed costs will increase to \$3,000 because of increases in depreciation and insurance costs. However, his variable costs will fall to \$25 per acre from the present \$30. What are the effects upon the break-even point and upon Tommy's expected profit? (Recall that Tommy planned to operate at a level of activity of 500 acres of production during the summer).

Answer:

1.  $BE_{units} = FC \div CM_{per\ unit} = \$3,000 \div (\$50 - \$25) = \underline{120\ acres}$ .
2. Expected profits at 500 acres of production are now:
  - a.  $500\ acres \times \$25\ CM_{unit} = \$12,500$  total contribution margin.
  - b.  $\$12,500\ CM_{total} - \$3,000\ Fixed\ Costs = \underline{\$9,500\ profit}$ .

By substituting fixed costs for variable costs, Tommy has increased his profits for the expected level of activity and has almost reached his desired \$10,000. However, if he chooses to do this, he also raises his break-even point from 100 units to 120. This means that his operations become a little more risky, since it will be harder to cover the fixed costs and begin making profits if the break-even point is raised. It will also be easier to incur losses if sales fall off.

- E. The **margin of safety** is a simple way to evaluate the risk that losses will result from falling sales levels. The margin of safety is a measure of the "breathing room" the company's current level of operations provides above and beyond the break-even point. The further beyond the break-even point the company operates, the more breathing room there is (and the smaller the chance that the company will fall down to it and begin to incur losses). Stated in units, it is simply the difference between the current activity level and the break-even point.

$$\mathbf{Margin\ of\ Safety}_{units} = \mathbf{Current\ Activity}_{units} - \mathbf{BEP}_{units}$$

The **margin of safety** can also be stated in terms of sales dollars:

$$\mathbf{Margin\ of\ Safety}_{sales} = \mathbf{Current\ Activity}_{sales} - \mathbf{BEP}_{sales}$$

Or, it can be stated as a percentage:

$$\text{Margin of Safety \%} = (\text{Current Activity}_{\text{sales}} - \text{BE}_{\text{sales}}) \div \text{Current Activity}_{\text{sales}}$$

or

$$\text{Margin of Safety \%} = (\text{Current Activity}_{\text{units}} - \text{BE}_{\text{units}}) \div \text{Current Activity}_{\text{units}}$$

*More Examples!* Recall that Tommy, in the discussions above, planned to operate at 500 acres of production during the summer. His original break-even point was 100 acres. This means he was originally going to operate at a level of activity that was 400 units beyond his breakeven point (or 400% beyond break-even). When he changed his cost structure in the previous example, the break-even point became 120 units. The change in his cost structure caused his *margin of safety* to fall by 20 units. He will now operate 380 units above the break-even point instead of 400 units. Previously, his level of production could have fallen by 80% (400/500) before he would begin to incur losses. Now if his sales level falls by 76% (380/500) he will begin to incur losses. Of course, the rewards (the additional profits) are probably well worth this slight increase in risk. The new margin of safety percent is calculated as follows:

$$\text{Margin of Safety \%} = (500 \text{ units} - 120 \text{ units}) \div 500 \text{ units} = 76\%$$

- E. The *degree of operating leverage* (or *DOL*) is an application of C-V-P analysis that involves determining the effects upon profits of anticipated changes in production levels. It is a ratio that relates total contribution margin at a given level of activity to net income:

$$\text{DOL} = \text{Total Contribution Margin} \div \text{Net Income}$$

- Note that the only difference between total contribution margin and net income is fixed costs:

$$\text{Total Contribution Margin} - \text{Fixed Costs} = \text{Net Income}$$

- Therefore, the greater the fixed costs, the larger the difference between  $\text{CM}_{\text{total}}$  and Net Income - and the greater the DOL ratio value. In general, a large DOL value means the company is substituting fixed costs for variable costs in its cost structure, just as Tommy did in the earlier example. When this happens, the result is often a large increase in profit (as happened in Tommy's case). However, the result is also increased volatility in profit (both on the upside and the downside) if activity levels change.
- In fact, there is an exact mathematical relationship between DOL and changes in profit. If the DOL is, say, 5.0, then the percentage change in profit will be 5 times as large as any percentage change in the activity level. That is, a 10% increase in unit sales will



produce a 50% (5 x 10%) increase in profits. A 20% decrease in unit sales will produce a 100% (5 x 20%) decrease in profits.

*Here's an Example!* In the earlier illustration Tommy changed his cost structure, substituting fixed costs for variable costs. As a result he achieved an increase in profits at his expected activity level of 500 units of production as follows:

<u>Original Expected Profit</u>		<u>New Expected Profit</u>	
Sales(500 acres @ \$50)	\$25,000	Sales(500 acres @ \$50)	\$25,000
Variable Cost(500 @ \$30)	(15,000)	New Variable Cost(500 @ \$25)	(12,500)
Contribution Margin	10,000	Contribution Margin	12,500
Fixed Costs	(2,000)	New Fixed Costs	(3,000)
Net Income	\$ 8,000	Net Income	\$ 9,500

Increase in Net Income: 19%

Tommy's DOL changed when the cost structure changed (though not by much):

Original DOL	New DOL
$\$10,000 \div \$8,000 = 1.25$	$\$12,500 \div \$9,500 = 1.316$

This means that the volatility in profit that would result from changes in activity has increased. If Tommy's business falls off and only 400 acres are mowed (a 20% decrease in activity), we should expect a decrease in profits now of 26.3% (20% x 1.316) instead of the 25% (20% x 1.25) he would have experienced before. Here is the proof:

<u>Profit @ 500 Acres</u>		<u>Profit @ 400 Acres</u>	
Sales(500 acres @ \$50)	\$25,000	Sales(400 acres @ \$50)	\$20,000
Variable Cost(500 @ \$25)	(15,000)	Variable Cost(400 @ \$25)	(10,000)
Contribution Margin	12,500	Contribution Margin	10,000
Fixed Costs	(3,000)	Fixed Costs	(3,000)
Net Income	\$ 9,500	Net Income	\$ 7,000

Percentage Decrease in Net Income: 26.3%

Of course, *increases* in activity (say 20% to 600 units) will result in greater *increases* in profit also (by 26.3% instead of 25%). So while volatility (i.e., risk) has been increased, so have potential profits.

*Note: In physics, "leverage" refers to the use of a small force to move a large object. In finance and accounting the concept is the same. A small change in activity now generates a larger change in profit. Therefore, profits have been "leveraged" through changes in the cost structure. The company's degree of operating leverage has increased.*

## V. Multiple Products, Sales Mix and CVP.

- A. We have assumed so far that the company has only one product, so there has been only one selling price and one variable cost amount to worry about. How can the analysis methods presented above be applied in a more realistic setting where the company may have several different products, each with its own selling price and variable cost?

- B. Two approaches may be taken to deal with this problem. One involves the contribution margin ratio and the other is a more straight-forward method that deals with units rather than sales.

*Here's an Example!* Suppose a manufacturer produces three products: A, B and C. The selling prices and variable costs of manufacturing each is presented below:

	A	B	C
Unit Selling Price	\$50	\$30	\$40
Variable Cost per Unit	(30)	(20)	(10)
Contribution Margin per Unit	\$20	\$10	\$30

*1. A Sales Mix Approach to Break-Even*

Fixed costs total \$190,000. How can a break-even point be found for this manufacturer? The key to finding the answer is to determine what the "sales mix" is, or, in other words, what proportion of total sales each of the products represents. Suppose it is known that A makes up 50% of total sales, B represents 30%, and C accounts for the remaining 20% of sales. This means that of every 10 units sold, 5 are A's, 3 are B's and 2 are C's. Now for a trick. Think of sales occurring in "packages" of units.

Each "package" represents 10 units (5 A's, 3B's and 2C's). The selling price for a "package" of production is \$420 (5x\$50 + 3x\$30 + 2x\$40). The variable cost of each "package" is \$230 (5x\$30 + 3x\$20 + 2x\$10). Therefore, the contribution margin for each package is \$190 (\$420 - \$230). The break-even point in terms of the number of packages that must be sold to cover the fixed costs is:

$$BE_{\text{in packages}} = \$190,000 \text{ FC} \div \$190 \text{ CM}_{\text{per package}} = \underline{1,000} \text{ "packages"}$$

Since each package is composed of 5 A's, 3B's and 2C's; then 5,000 A's, 3,000 B's and 2,000 C's must be sold in order to break even. Also, since the selling price of a package is \$420; then the break-even point in sales dollars is \$420,000 (\$420 x 1,000 packages).

*2. A Contribution Margin Ratio Approach to Break-Even*

Again, this approach relies upon a constant sales mix, but it is not necessary to determine what the sales mix is. Instead we only need to know what the total sales, total variable cost, and the fixed costs are. For example, assume that the manufacturer above had produced and sold 10,000 A's, 6,000 B's and 4,000 C's in the previous period (note that the sales mix here is again 50% A, 30% B and 20% C). This would produce the following income statement:

Sales (10,000x\$50 + 6,000x\$30 + 4,000x\$40)	\$840,000
Variable Costs (10,000x\$30 + 6,000x\$20 + 4,000x\$10)	(460,000)
Total Contribution Margin	380,000
Fixed Costs	(190,000)
Net Income	\$190,000

From this income statement, the overall contribution margin ratio can be calculated:

$$CMR_{\text{total}} = \$380,000 \text{ CM}_{\text{total}} \div \$840,000 \text{ Sales} = .4524$$

The break-even point in sales can then be calculated as follows:

$$BE_{\text{sales}} = \$190,000 \text{ FC} \div .4524 \text{ CMR} = \underline{\$420,000}.$$

Note that while this does provide a break-even point, it does not allow us to determine the number of units of A, B and C that must be sold in order to break even. That information could be useful in managing operations.

**VI. Assumptions Inherent in CVP Analysis.** In our analyses in this module, we have made several assumptions that should be understood before attempting to apply the procedures in practice. They are as follows:

- All costs can be classified as fixed or variable, behave accordingly, and can therefore be predicted accurately.
- The relationship between selling price and cost is constant. That is, if costs rise in the period being analyzed, the selling price will be increased to preserve the contribution margin per unit so that the predicted results are still valid.
- Production efficiency is constant through the relevant range of production (so that cost behavior remains constant).
- Total fixed costs, variable cost per unit, and selling price per unit are constant throughout the relevant range.
- The sales mix remains as predicted (in the case of multi-product CVP).
- Production and sales occur evenly (no inventory build-up or draw-down).

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### ***Summary***

#### ***Visual Fit, High-Low, and Linear Regression Illustration***

#### ***Linear Regression and Excel***

This material is supplemental to the course content, and you will not be tested directly over linear regression in this course. It is presented here because *least-squares regression* is easy to apply and it is commonly utilized in industry. The *high-low method* is also illustrated, and you are required to be able to apply it on examinations and homework in this course.

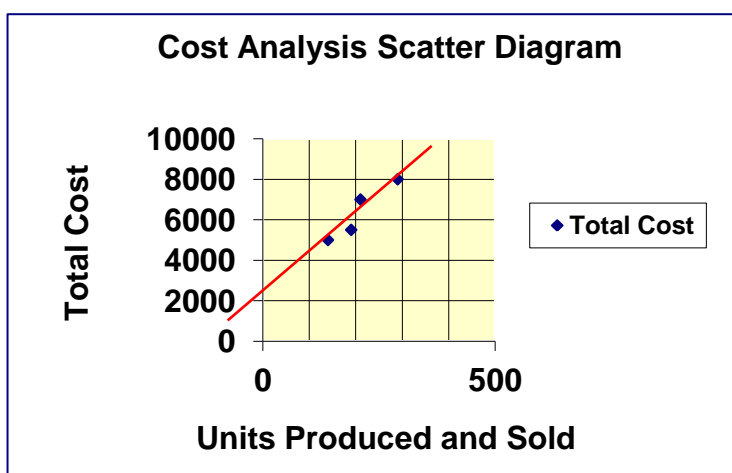
The least-square method may be applied algebraically (the long-hand method), or it may be done electronically using either a spreadsheet program (such as Microsoft Excel) or a calculator with statistical functions.

The section below details the process required to solve for variable and fixed costs using the linear regression method in Excel. You are not responsible for being able to enter data into Excel and calculating variable and fixed costs, but you should be able to understand how Excel can be used to provide the best estimates possible for variable and fixed costs.

In the example, we are given utilities cost information. Since some power is consumed even when nothing is being produced, utilities would represent a mixed cost. We will use various methods, including linear regression techniques, to analyze this mixed cost and determine its variable and fixed cost components. Assume that utility costs have been recorded as shown below for the last 4 quarters of the year:

<u>Period</u>	<u>Utilities Cost</u>	<u>Units Produced</u>
1	\$8,000	290
2	\$7,000	210
3	\$5,000	140
4	\$5,500	190

The scatter diagram below has been prepared, and a total cost line has been sketched on the graph using the visual fit method (this is the red line in the illustration). According to this rough approximation, the fixed costs appear to be about \$2,000 and the variable cost per unit is around \$20 per unit [(\$8,000 - \$2,000)/300 units]. However, because the points do not plot on a straight line and there is error in the visual fit estimate, management has decided to apply the linear regression method to get more accurate results.

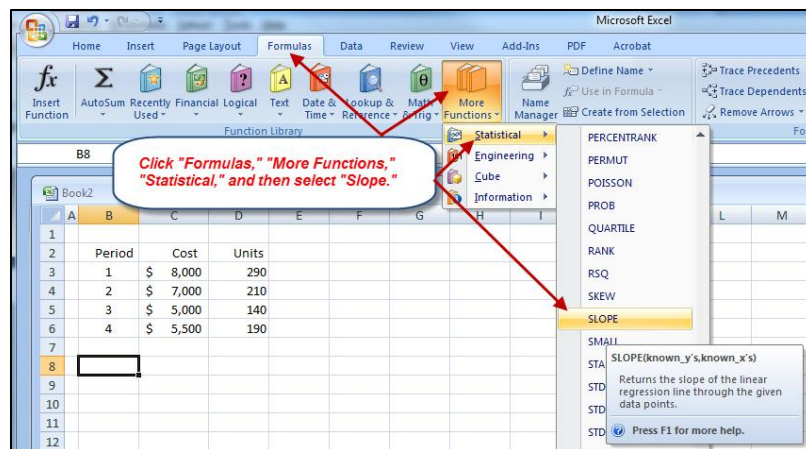


1. To use Excel in solving this problem, we need to set up the data in columns. This is illustrated below.

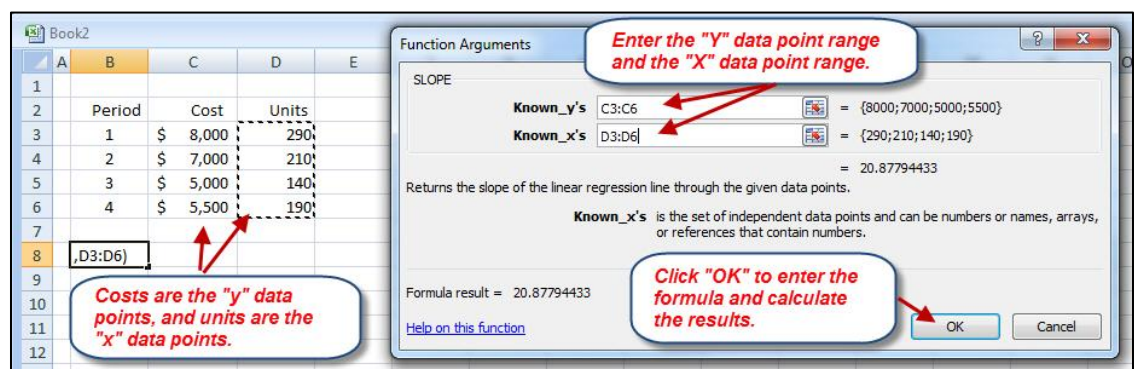
	A	B	C	D
1				
2		Period	Cost	Units
3		1	\$ 8,000	290
4		2	\$ 7,000	210
5		3	\$ 5,000	140
6		4	\$ 5,500	190
7				

2. We next use the Excel "Slope" formula to calculate the slope of the regression line. This will give us our variable cost per unit. Remember, the regression line is the mathematically determined line that best fits all the data points in our sample, so the slope of the regression line will represent the most accurate estimate of the variable cost per unit that it is possible to obtain.

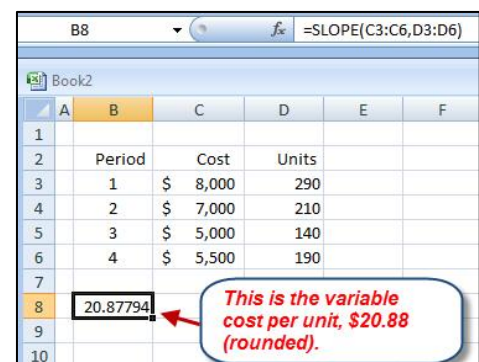
Click on the cell in which you want to insert the formula (cell B8 is used below), and then click the “Formulas” tab in Excel. Select the “SLOPE” function from the list of available functions (see below).



- When the SLOPE formula is clicked, the data entry box shown below appears. To calculate the variable cost component of this mixed cost it is necessary to enter all of our data points into the Excel formula. Do this by entering the range of the cost amounts (cells C3 through C6) as the “Y” values, and then entering the range of unit values (cells D3 through D6) as the “X” values.

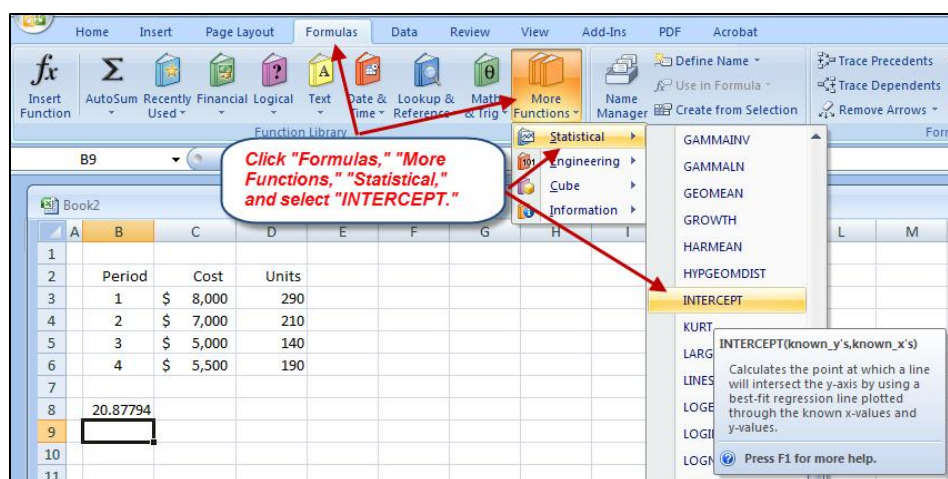


- Click “OK” and the SLOPE value, \$20.87794 appears in the spreadsheet (see opposite). We will round this to \$20.88 per unit, and use it as the variable cost per unit in all of our CVP calculations. Remember, this is the very best estimate for variable cost per unit that we can obtain, since the SLOPE formula is mathematically precise. Our visual fit estimate, \$20, was pretty close to this amount, but it was still off by 88 cents per unit (enough of an error to perhaps result in poor management decisions).
- We next use the Excel “Intercept” formula to calculate the intercept of the regression line, the point at which it crosses over the

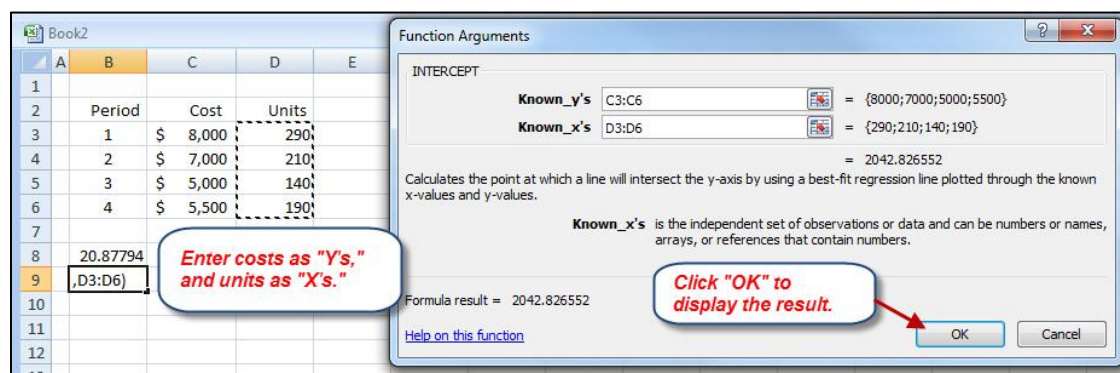


cost axis when the number of units is zero. This will give us our total fixed cost amount. Remember, the regression line is the mathematically determined line that best fits all the data points in our sample, so the intercept of the regression line will represent the most accurate estimate of the total fixed costs that it is possible to obtain.

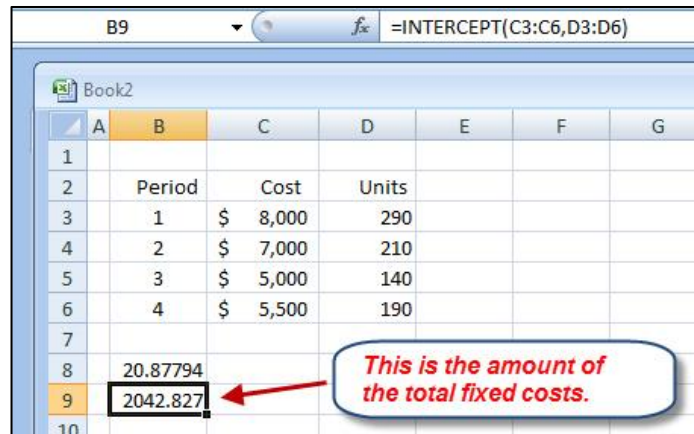
Click on the cell in which you want to insert the formula (cell B9 is used below), and then click the “Formulas” tab in Excel. Select the “INTERCEPT” function from the list of available functions (see below).



- When the INTERCEPT formula is clicked, the data entry box shown below appears. Once again, it is necessary to enter all of our data points into the Excel formula. Do this by again entering the range of the cost amounts (cells C3 through C6) as the “Y” values, and then entering the range of unit values (cells D3 through D6) as the “X” values.



- Click “OK” and the INTERCEPT value, \$2,042.827 appears in the spreadsheet (see below). We will round this to \$2,042.83, and use it as the total fixed cost amount in all of our CVP calculations.



8. Remember, this is the very best estimate for total fixed costs that we can obtain, since the INTERCEPT formula is also mathematically precise. Our visual fit estimate, \$2,000, was pretty close to this amount, but it was still off by \$42.83 per unit (enough, again, to perhaps result in poor management decisions).

### SUMMARY OF REGRESSION RESULTS

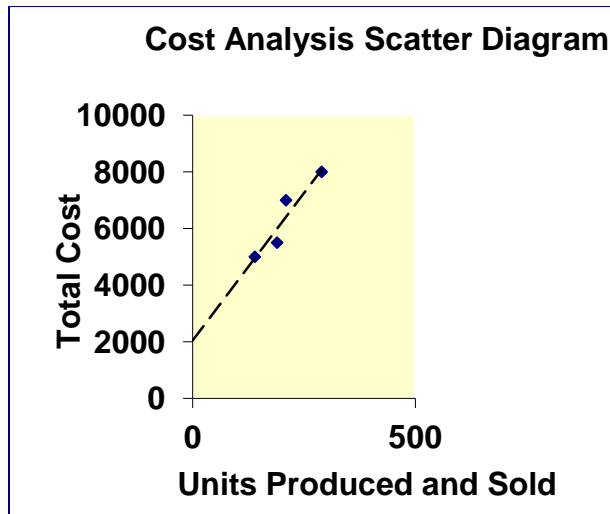
Would you like to check our results and see how well they fit the data? One way to do that would be to use the results above to calculate a “predicted” cost for the units in the sample. Since the computed fixed cost is \$2,042 and the variable cost is \$20.88 per unit, the predicted total cost for any number of units would be equal to \$2,042 in fixed costs plus \$20.88 times the number of units produced (which would equal total variable cost):

$$\text{Total Predicted Cost} = FC + (VC_{\text{unit}} \times \text{Units Produced})$$

This formula is a good planning tool, since it allows us to predict future costs for budgeted levels of production and sales. It is very useful in budgeting. Applying this formula to our earlier production levels gives us the following predicted costs:

<u>Period</u>	<u>Predicted Cost</u>	<u>Units Produced</u>
1	\$8,097	290
2	\$6,427	210
3	\$4,966	140
4	\$6,010	190

If you compare these with the actual costs that were presented earlier, you can see that the predicted costs are very close to the actual ones. The graph below shows the original data points and plots the “regression line,” the line of best fit, against them.



Each of the predicted values in the table above represents a point on the regression line. The points that lie off the line are the actual cost figures given for the number of units indicated. The distance between the points and the regression line represents the error in our estimates, and since they are so small it appears that the regression line does represent the actual data very well.

Another statistical measure, the *correlation coefficient*, may be used to mathematically measure just how well the regression line “fits” the actual data points. We may calculate it by selecting “CORREL” from the list of statistical formulas. When we enter our data points, the value that results is .946, or 94.6%.

The screenshot shows an Excel spreadsheet with the following data:

Period	Cost	Units
1	\$ 8,000	290
2	\$ 7,000	210
3	\$ 5,000	140
4	\$ 5,500	190

The CORREL function is being applied to the data ranges C3:C6 and D3:D6. The function arguments dialog box shows the following details:

- Array1: C3:C6 = {8000;7000;5000;5500}
- Array2: D3:D6 = {290;210;140;190}
- Formula result = 0.945922324

A callout box states: "Applying the CORREL formula gives us a result of .9459, or 94.6%."

This means that 94.6% of the variance in the data points is explained by the regression line. In other words, the regression line really does fit the data well. A value of zero would mean there is no correlation between the data and the line, and a value of 100% would indicate that there is no error at all in the estimates. Our correlation is not perfect, but since it is so high we should have a high degree of confidence in our calculated fixed and variable costs.



## High-Low Method

Were we to calculate the variable and fixed cost components of the utilities cost using the high-low method, we would first determine the high and the low points:

1	\$8,000	290
2	\$7,000	210
3	\$5,000	140
4	\$5,500	190

We would then use them to determine the variable cost per unit and the fixed costs.

	Cost	Units
High Point	\$8,000	290
Low Point	\$5,000	140
Changes in Cost and Quantity	\$3,000	150
Estimated Variable Cost = \$3,000 / 150 units =	\$20 per unit	
Estimated Fixed Cost = \$8,000 – (\$20 x 290) =	\$2,200	
<i>(or, alternatively, Fixed Cost = \$5,000 – (\$20 x 140) = \$2,200.)</i>		

Note that these results are close to the “best fit” estimates obtained using linear regression, but we cannot expect to come so close every time. The high-low method must be applied with caution.

**-END-**